Parametric resonance of a shallow arch microbeam for sensing applications

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Summary. In this work a clamped-guided shallow arch microbeam is considered. The actuation of the microbeam is both through a transverse and axial electrostatic forces. The considered equation of motion takes into account the initial rise of the microbeam and the sliding degree of freedom at the guided side. The static and free vibration problems are solved using the Differential Quadrature Method. The results are validated with previously published experiments found in the literature. The model is able to reproduce experimental findings regarding fundamental resonance and parametric resonance of the first mode.

Introduction

Parallel-plate based MEMS actuators are limited by the occurrence of pull-in in the static and dynamic regimes. An example of design improvement was proposed by Abu-Salih and Elata [1], where they introduced a curved microbeam obtained by buckling. Ouakad and Younis [2] proposed to analyze the nonlinear dynamics of curved microbeam using perturbation techniques. They concluded that the initial deflection of the curved beam strongly affects its behavior. Therefore, it can be used as control parameter to tune the response of the device. Recently, Ramini et al. [3] developed and experimentally tested a design in which the initial gap of the arched microbeam can be tuned using a transverse electrostatic parallel plate actuator. The design allows also the introduction of a parametric excitation through the applied axial force. Excitations at secondary resonances generate large stroke actuation generally unachievable by classical MEMS actuators. The device fabricated and tested by Alcheikh et al. [4], depicted high tunability up to 160% for the first and third mode shapes. Ouakad et al. [5] demonstrated that three-to-one and one-to-one internal resonance are achievable by controlling the initial rise of the arched microbeam. The proposed work aim to model an arched microbeam with an applied electrostatic axial force. The proposed mathematical model should reproduce the experimental finding of the previously fabricated shallow arch microbeams.

Problem Formulation

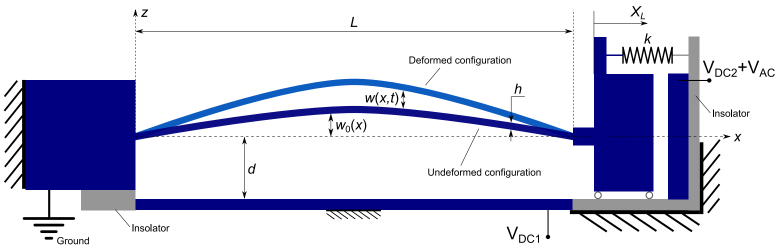

Figure 1: Clamped-guided shallow arch microbeam with transverse and axial excitations.

We consider a clamped-guided shallow arch given by Figure 1, of initial shape $w_0(x)$, width b, thickness h, length L, modulus of elasticity E, cross sectional are $A = bh$, moment of inertia I, mass density ρ and M and k are the mass and stiffness of the moving side electrode. The axial displacement is denoted by $u(x,t)$ and the transverse displacement is denoted by $w(x,t)$ and assumed to be around the initial curvature $w_0(x)$ (Figure 1). Using the Euler-Bernoulli beam theory and the von Karman nonlinear strain, the extended Hamilton's principle is used to derive the governing equations and boundary conditions. They are given as follows:

$$\rho A\ddot{w} + EIw'''' + c\dot{w} = N_L\left(w'' + w_0''\right) + \frac{\epsilon b V_1^2}{2(d+w+w_0)^2} \tag{1}$$

$$M\,\ddot{X}_L + kX_L = \frac{\epsilon A_{pp}(V_2 + V_{AC}\cos(2n\pi f t))^2}{2(g - X_L)^2} - N_L \tag{2}$$

$$w(0,t) = 0, \quad w'(0,t) = 0, \quad w(L,t) = 0, \quad w'(L,t) = 0 \tag{3}$$

$$N_L(t) = \frac{EA}{L}\left[X_L + \frac{1}{2}\int_0^L \left(w'^2 + 2w'w_0'\right)dx\right] \tag{4}$$

where $n = 1$ or 2 for the fundamental and parametric excitations, respectively.
Equations 1 to 4 are solved using the Differential Quadrature Method (DQM) for the space derivative and then the Finite Difference Method (FDM) is used to calculate limit cycle solution [6].

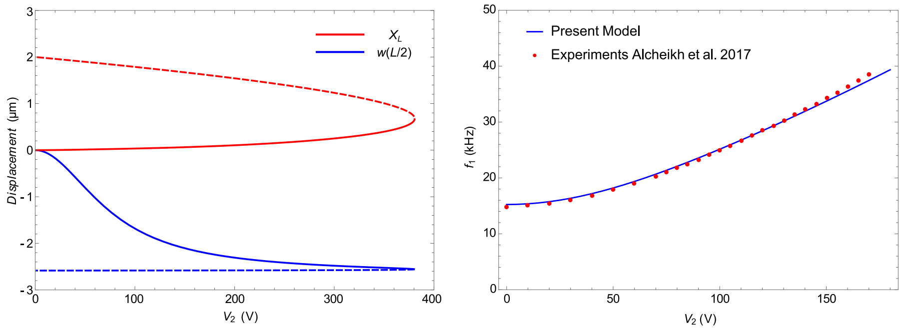

Figure 2: (a) Variation of the static deflection $w(L/2)$ with respect to V_2 for $d_0 = 2.6\mu$m. (b) Variation of the first natural frequency for $d_0 = 2.6\mu$m.

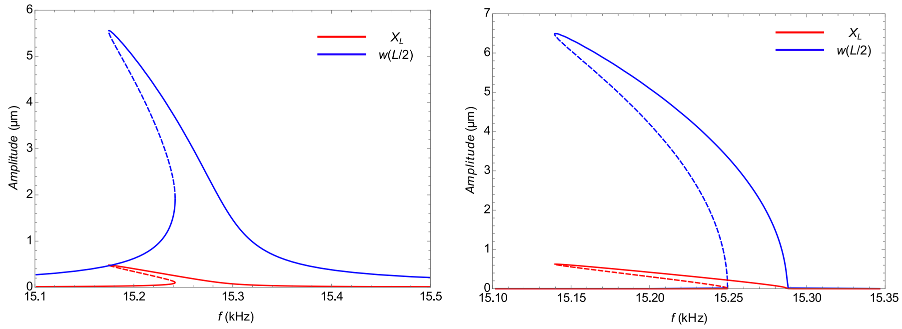

Figure 3: Frequency-response curves of the midspan transverse deflection and axial displacement for $V_2 = 3$V and $V_{AC} = 5$V when $d_0 = 2.6\mu$m and $f_1 = 15.27$kHz. (a) Fundamental resonance, (b) Principal parametric resonance of the first mode.

Static and free vibration of the microbeam

The static response of the microbeam is shown in Figure 2-a. Only the effect of the side voltage V_2 has been considered here. As shown, the static response of the transverse deflection at midspan $w(L/2)$ is highly constrained when the microbeam recover a straight position at $w(L/2) = d_0$. The axial displacement at $x = L$, expressed by X_L, is responding as a classical electrostatic parallel plate actuator. The dashed branches denote unstable solutions. The free vibration problem is also solved under the effect of the applied side voltage V_2. The results shown in Figure 2-b are compared with those obtained experimentally by Alcheikh et al. [4]. As shown, very good agreement is obtained.

Fundamental resonance and parametric resonance of the first mode

The frequency-response curves for the fundamental resonance are shown in Figure 3-a for the midspan transverse deflection and axial displacement. A softening behavior is deduced from the simulations. A similar behavior is observed for the parametric resonance of the first mode in Figure 3-b. Also here, dashed branches denote unstable solutions.

Conclusions

We proposed a nonlinear dynamic analysis of a clamped-guided shallow arch microbeam. The governing equations and associated boundary conditions are derived and the solved using a combination of Differential Quadrature Method, for the space derivative, and the Finite Difference Method for the limit-cycle solutions. The static and free vibration responses have been simulated for the transverse and side displacements. The variation of the fundamental frequency with respect to the applied side voltage is compared and validated with published experimental results. The frequency-response curves showed that both fundamental and parametric resonances are possible in accordance with previously reported experimental analyses.

References

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