Hybrid formalism for consensus of nonholonomic robots with biased measurements*

T. Borzone*,†, I.-C. Morărescu*, M. Jungers*, M. Boc† and C. Janneteau†
* *T. Borzone, I.-C. Morărescu and M. Jungers are with Université de Lorraine, CNRS, CRAN, F-54000 Nancy, France,*
† *T. Borzone, M. Boc and C. Janneteau are with CEA LIST (Commissariat à l'Energie Atomique et aux Energies alternatives - Communicating Systems Laboratory)*

Summary. This work focuses on the design of a control strategy for consensus of fleets of nonholonomic robots in presence of noisy measurements. The proposed strategy leads to a hybrid behaviour of the closed-loop dynamics of each robot. Precisely, the robots sporadically compute references that are kept constant until the next update. Between two updates of the reference the robots will apply a tracking controller allowing them to approach exponentially fast their targets. The main contributions are related to both the control design and the analysis of the resulting hybrid closed-loop dynamics. Simulations and real experiments will be presented at ENOC.

Introduction

During the last decades the design of control laws for nonholonomic robots received a lot of attention. This is mainly due to the fact that Brockett conditions [2] required for smooth stabilisation do not hold for this class of dynamics. To overcome the fact that smooth state-feedback controllers stabilizing the position and orientation of the robot do not exist, different discontinuous control laws have been proposed [5, 1]. We also point out that a control design for tracking a smooth trajectory has been also proposed for nonholonomic robots [4]. It is noteworthy that, all these strategies consider a stand-alone robot that is continuously controlled. *Our objective is to control a fleet of nonholonomic robots in a decentralized manner and in a harsh environment hampering continuous communications between robots.*

The main contribution of our work is twofold: first, the design of a hybrid control strategy for consensus of nonholonomic robots and second, the stability analysis of the closed-loop system providing minimum dwell-time conditions guaranteeing the overall system stability. As shown in the Figure 1 below, our strategy contains two loops. The external loop takes place in discrete time and uses local information related to neighbouring robots state in order to sporadically compute references of the robots. The internal loop takes place in continuous time, does not require communication with (or information from) other robots and uses a standard tracking (or point stabilization) controller such as the one proposed in [4].

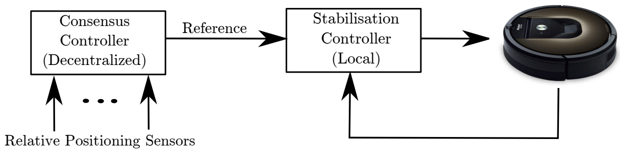

Figure 1: Control structure

Problem formulation

We consider a fleet of n nonholonomic robots that have to reach a consensus in the positions without requiring specific final orientation of the agents. For the sake of simplicity we remove the time argument t when it is not explicitly needed. We denote by $r_i = (r_{x_i}, r_{y_i})$ the 2D reference position for the robot i and we fix $r_{\theta_i} = 0$ its heading reference. The Cartesian coordinates of the center of mass of each vehicle with respect to the fixed inertial frame are denoted using vector $X_i = (x_i, y_i)$. Denoting $e_i = (e_{x_i}, e_{y_i}, e_{\theta_i})^\top$ the dynamics of the i^{th} robot is described by the following differential equations

$$\dot{e}_i = g(e_i)u_i, \quad g(e_i) = \begin{bmatrix} \cos e_{\theta_i} & 0 \\ \sin e_{\theta_i} & 0 \\ 0 & 1 \end{bmatrix}, \quad u_i = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}. \tag{1}$$

where v_i is the linear velocity and ω_i is the angular velocity of the mobile robot; e_{x_i} and e_{y_i} are the Cartesian coordinates of the center of mass of the vehicle with respect to a frame positioned on the reference position r_i, and e_{θ_i} is the angle between the heading direction and the x-axis of this frame.

The point stabilization control considered in this work is the continuous piecewise smooth control law introduced in [3]. Basically, one considers a map $F : \mathbb{R}^3 \longmapsto \mathbb{R} \times (-\pi, \pi]$ relating $e_i \in \mathbb{R}^3$ to $z_i = (a_i, \alpha_i)^\top \in \mathbb{R} \times (-\pi, \pi]$. Taking $K, \gamma > 0$ the control law $u_i = \kappa(e_i) = (-\gamma b_1(e_i)a, -b_2(e_i)v - Ka)^\top$, where b_1, b_2 are explicitly defined in [3], exponentially stabilizes the origin of the planning reference frame $e_i = 0$. In the following, we denote $\varepsilon_i = (e_{x_i}, e_{y_i})$ the 2D Cartesian error coordinates *i.e.* $\varepsilon_i = X_i - r_i$.

Lemma 1 *Let us consider the closed loop dynamics $\dot{e}_i = g(e_i)\kappa(e_i)$ with $\kappa(e_i)$ defined above. Then, there exist positive constants c_ε and λ_ε such that $\forall t \geq t_0$ one has $\|\varepsilon_i(t)\|_\infty \leq \sqrt{n}c_\varepsilon \|\varepsilon_i(t_0)\|_\infty \mathrm{e}^{-\lambda_\varepsilon(t-t_0)}$, $\forall i \in \{1, \ldots, n\}$.*

*This work was partially funded by the Région Grand Est and CEA Tech Metz

As we previously said, the external loop designs references for each robot based on the sensing of relative positions of some neighbouring robots. These "interactions" are mathematically captured by a time-varying digraph (directed graph) $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$, where the vertex-set $\mathcal{V}$ represents the set of robots and the edge set $\mathcal{E}(t) \subset \mathcal{V} \times \mathcal{V}$ collects the interactions between robots at time t. A *path of length* p in a digraph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ is a union of directed edges $\bigcup_{k=1}^{p}(i_k, j_k)$ such that $i_{k+1} = j_k$, $\forall k \in \{1, \ldots, p-1\}$. The node j is *connected* with node i in $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ if there exists at least a path in $\bar{\mathcal{G}}$ from i to j (*i.e.* $i_1 = i$ and $j_p = j$). A *strongly connected digraph* is such that any of its two distinct elements are connected. In the sequel we consider the set of instants when at leat one robot updates its reference as $\mathcal{T} = \{t_k : t_k \in \mathbb{R}^+, t_k < t_{k+1}, \forall k \in \mathbb{N}, \lim_{k\to\infty} t_k = \infty\}$. Moreover, we define $\mathcal{T}_i \subset \mathcal{T}$ selecting the instants when robot i updates its reference (*i.e.* $\mathcal{T} = \bigcup_{i=1}^{n} \mathcal{T}_i$). At each instant $t_k \in \mathcal{T}$ a graph structure $\mathcal{G}(t_k) = (\mathcal{V}, \mathcal{E}(t_k))$ defines the interactions between neighbours. Let $\alpha \in (0,1)$, $\beta \in (1/2, 1)$ be given. To each graph structure $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ we uniquely associate a row stochastic matrix P satisfying the following properties: $P_{i,j} = 0$, if $(i,j) \notin \mathcal{E}$, $P_{i,i} > \beta$, $\forall i = \{1, \cdots, n\}$, $P_{i,j} > \alpha$, if $(i,j) \in \mathcal{E}$.

Assumption 1 (Connectivity) *The digraph* $\mathcal{G} = \bigcup_{k\geq k_0} \mathcal{G}(t_k)$ *is strongly connected for all* $k_0 \in \mathbb{N}$.

Assumption 2 (Bounded Intercommunication Interval) *If* $(i,j) \in \mathcal{E}(t_k)$ *infinitely often, then there is some* $L \in \mathbb{N}$ *such that, for all* $t_k \in \mathcal{T}$, $(i,j) \in \mathcal{E}(t_k) \bigcup \mathcal{E}(t_{k+1}) \bigcup \cdots \bigcup \mathcal{E}(t_{k+L-1})$.

Main results

The closed-loop hybrid dynamics associated with the i-th robot ($i \in \{1, \ldots, n\}$) can be formulated as:

$$\begin{cases} \dot{r}_i(t) = 0, \qquad \dot{e}_i(t) = g(e_i(t))\kappa(e_i(t)) \quad \text{for } t \in \mathbb{R}^+ \setminus \mathcal{T}_i, \\ r_i(t_k) = \displaystyle\sum_{j=1}^{n} P_{i,j}(t_k)\varepsilon_j(t_k^-) + \sum_{j=1}^{n} P_{i,j}(t_k) r_j(t_k^-) + \delta_i(t_k), \quad \varepsilon_i(t_k) = \varepsilon_i(t_k^-) + r_i(t_k^-) - r_i(t_k) + \delta_i(t_k) \text{ for } t_k \in \mathcal{T}_i, \end{cases}$$

where $\delta_i(t_k)$ represents the measurement bias whose infinity norm is upper-bounded by $\bar{\delta} \in \mathbb{R}^+$. Our main objective is to analyse the ISS property of the overall dynamics of n robots defined above w.r.t the following set:

$$\mathcal{A} = \Big\{\varepsilon, r \in \mathbb{R}^{2n} \mid \varepsilon = 0,\ r_{x_i} = r_{x_i},\ r_{y_i} = r_{x_j}, \forall i,j \in \{1, \ldots, n\}\Big\}.$$

Theorem 1 *Let Assumptions 1-2 hold. Then the overall dynamics of* n *systems defined above is ISS w.r.t. the set* $\mathcal{A}$, *if the time between any two consecutive updates of the reference for each robot is larger than* τ^* *with*

$$\tau^* = \frac{1}{\lambda} \ln \max \left\{ \frac{\eta_2 c N}{1-\eta_1} \sum_{l=0}^{2L-1} a^{\frac{2L-1-l}{2L-1}}, \frac{c(\eta_3(N-1)+1)}{(1-n(2L-1)\eta_3)a} \sum_{l=0}^{L-1} a^{\frac{2L-1-l}{2L-1}} \right\} > 0,$$

with $\eta_1 = 1 - \alpha^L, \eta_2 = 4 - 2\alpha, \eta_3 = 1 - \beta$ *and* $a = \frac{1+\eta_1}{2}$.

Remark 1
- *We note that theorem above does not fix the update instants but only provide a minimum dwell-time between updates of the reference of the same robot. Consequently, very frequent updates are forbidden.*
- *When* $\bar{\delta} = 0$ *(i.e. the sensors provide perfect measurements) the theorem above states that* $\mathcal{A}$ *is GUAS for the overall closed-loop dynamics of* n *nonholonomic robots.*

Conclusions

This work formulates a decentralized control strategy for fleets of nonholonomic robots with biased measurements. The control is designed as follows. In a first step, at sporadic time-instants the robots compute a reference based on measurements of relative postions of some neighbours. In a second step, the robots continuously apply a state-feedback tracking controller. This yields a hybrid behaviour of the closed-loop dynamics. We show that consensus can be achieved as far as a certain minimum dwell-time condition between the reference updates of each robot is respected. Simulations and real experiments will be presented at ENOC.

References

[1] A. Astolfi. Exponential stabilization of a wheeled mobile robot via discontinuous control. *Transactions-American Society of Mechanical Engineers Journal of Dynamic Systems, Measurement, and Control*, 121:121–125, 1999.

[2] R.W. Brockett. Asymptotic stability and feedback stabilization. *In Differential Geometric Control Theory*, (R.W. Brockett and R.S. Millman and H.J. Sussmann Eds):181–191, 1983.

[3] C. Canudas de Wit and O. J. Sordalen. Exponential stabilization of mobile robots with nonholonomic constraints. *IEEE Transactions on Automatic Control*, 37(11):1791–1797, 1992.

[4] E. Panteley, E. Lefeber, A. Loria, and H. Nijmeijier. Exponential tracking control of a mobile car using a cascaded approach. In *Proceedings of IFAC Motion Control*, pages 201–206, 1998.

[5] J.-B. Pomet. Explicit design of time-varying stabilizing control laws for a class of controllable systems without drift. *Systems & control letters*, 18(2):147–158, 1992.