Trajectory Tracking Control for Linear Complementarity Systems with Continuous Solutions

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Summary. This work concerns the trajectory tracking control for Linear Complementarity Systems (LCS) with continuous solutions. Such systems are strongly nonsmooth and nonlinear. The tracking issue is solved using passivity tools that yield conditions which can be solved with Linear Matrix Inequalities (LMI). Circuits with ideal diodes illustrate the theoretical developments.

1 Introduction

Trajectory tracking is a major problem in Automatic Control. It is well understood for linear time-invariant systems (see [1] and references therein) and some classes of nonsmooth systems [2, 3]. In this study, we study the LCS given by

$$\begin{cases} \dot{x}(t) = Ax(t) + B\lambda(t) + Eu(t), \\ 0 \le \lambda(t) \perp Cx(t) + D\lambda(t) + Fu(t) \ge 0, \\ x(0) = x_0, \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, \lambda(t) \in \mathbb{R}^p$ with $D = 0, D \succeq 0$, and $D \succ 0$. In case $D \succeq 0$, we restrict to positive semidefinite matrices D of the form $\begin{pmatrix} D_1 & 0 \\ 0 & 0 \end{pmatrix}$, where $D_1 \succ 0$ is square of dimension $q < p$. Our goal is to design the controller u such that $\|x(t) - x_d(t)\| \to 0$ as $t \to \infty$, where x_d is a desired state trajectory. Generally, to find a controller satisfying a given reference is a hard problem. Thus our ambition is only to deal with some sub-classes of problems which we can handle.

1.1 Main Results

Let us first make the following assumptions.

Assumption 1 *There exists a multiplier λ_d such that desired trajectory x_d satisfies*

$$\begin{cases} \dot{x}_d(t) = Ax_d(t) + B\lambda_d(t) + Eu_d(t) \\ 0 \le \lambda_d(t) \perp Cx_d(t) + D\lambda_d(t) + Fu_d(t) \ge 0, \end{cases}$$

for a given input $u_d \in L^1_{\text{loc}}(\mathbb{R}_+; \mathbb{R}^m)$.

Assumption 2 *There exists a matrix K such that the quadruple $(A + EK, B, C + FK, D)$ is strictly passive.*

Then the following result holds.

Proposition 1 *Suppose that Assumptions 1 and 2 hold. Then the closed-loop system (1) with the state feedback controller*

$$u(t) = K[x(t) - x_d(t)] + u_d(t)$$

has a unique global solution $x(\cdot)$, and $\|x(t) - x_d(t)\| \to 0$ as $t \to +\infty$.

The proof is led with the Lyapunov function $V(z) = z^\top Pz$, $z = x - x_d$, and $P = P^\top \succ 0$ is a solution of the passivity LMI [4, Lemma 3.16, Theorem 4.73]. The controller gain K is calculated by solving the LMI:

$$\begin{pmatrix} QA^\top + AQ + L^\top E^\top + EL + \varepsilon Q & B - QC^\top - L^\top F^\top \\ B^\top - CQ - FL & -(D + D^\top) \end{pmatrix} \preccurlyeq 0. \tag{2}$$

This gives an LMI feasibility problem in the new variables $Q = Q^T \succ 0$ and L. After solving this LMI, the feedback gain K can be recovered from $K = LQ^{-1}$. It may happen that the above LMI has no solutions (see [5, section 2.5.1] for an example). This can be solved by changing the controller structure. Namely, we allow for not only a state feedback, but also that the multiplier $\lambda(t)$ be measurable and part of the controller. In practice the multiplier may be voltages or currents (for circuits) or contact forces (for mechanical systems) and could be measured.

Assumption 3 *There exist matrices K, G such that the quadruple $(A + EK, B + EG, C + FK, D + FG)$ is strictly passive and $D + FG$ is either a zero matrix, or a positive definite matrix, or a matrix in the form $\begin{pmatrix} D_1 & 0 \\ 0 & 0 \end{pmatrix}$ with $D_1 \succ 0$.*

Proposition 2 *Suppose that Assumptions 1 and 3 hold. Then the closed-loop system (1) with the controller*

$$u(t) = K[x(t) - x_d(t)] + G[\lambda(t) - \lambda_d(t)] + u_d(t)$$

has a unique global solution $x(\cdot)$ and $\|x(t) - x_d(t)\| \to 0$ as $t \to +\infty$.

2 Example and Simulations

Let us consider the circuit in Figure 1 with an ideal diode, having the dynamics: $\begin{cases} \dot{x_1}(t) = x_2(t) \\ \dot{x_2}(t) = -\frac{1}{\mathbf{LC}}x_1(t) + \frac{\lambda(t)}{\mathbf{L}} + \frac{u_2(t)}{\mathbf{L}} \\ 0 \leq \lambda(t) \perp \frac{\lambda(t)}{\mathbf{R}} + x_2(t) - \frac{u_1(t)}{\mathbf{R}} \geq 0. \end{cases}$.

From Proposition 1, we get the controller $u^1 = K(x - x_d) + u_d$ with $K = \begin{pmatrix} -1.554224 & -0.261066 \\ -3.228662 & -3.663074 \end{pmatrix}$, and from Proposition 2, the controller $u^2 = K(x - x_d) + G(\lambda - \lambda_d) + u_d$ with $K = \begin{pmatrix} -2.833123 & -1.041382 \\ -4.568759 & -3.291980 \end{pmatrix}$, $G = \begin{pmatrix} -0.500000 \\ 5.121218 \end{pmatrix}$.

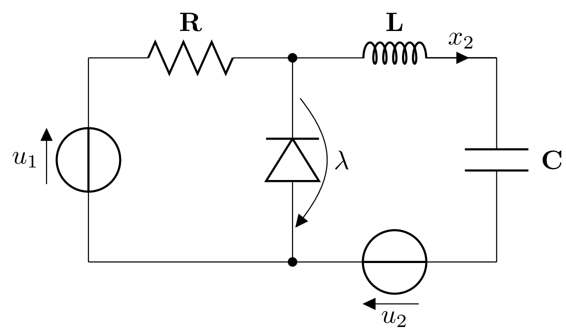

Figure 1: RLCD circuit with two voltage sources.

The matrices are obtained by using YALMIP (https://yalmip.github.io/) with 6-digit accuracy, and using the INRIA code SICONOS (https://nonsmooth.gricad-pages.univ-grenoble-alpes.fr/siconos/index.html) to get the numerical results of this problem. The results are depicted in Figure 2, where x_i^j is the ith component of the closed-loop system's state with controller u^j, and initial conditions $(0.5, 0.5)$.

3 Conclusions

A detailed presentation of this work is made in the report [5] where well-posedness issues are presented, as well as several circuits examples with simulations, and all the codes needed to compute the controller gains (MATLAB, YALMIP and SICONOS).

References

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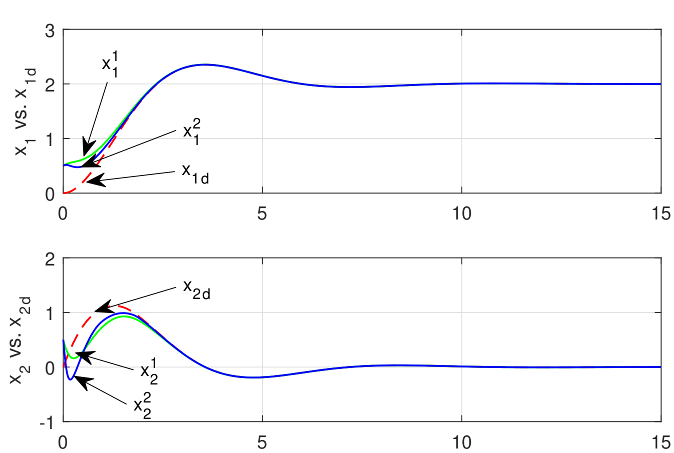

Figure 2: The desired trajectory and the state when using controller u^1 and u^2.