Topology of exotic wakes

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Summary. The formation of vortices in the flow around an oscillating cylinder is shown to be a complex set of topological bifurcations as the amplitude of the cylinder oscillations is varied. We show that the transition from a wake where two vortices are shed per cycle, to a configuration with three vortices are shed, goes through intermediate stages, where secondary vortices are created but disappear again.

Introduction

The wake behind a transversely oscillating cylinder allows a remarkable richness of flow structures. Vorticity created at the cylinder surface is advected downstream and organizes into distinct vortices. For a stationary cylinder in the periodic regime, two vortices of opposite circulation are shed per cycle forming the famous von Kármán vortex street. This is also denoted a 2S wake, where S means the shedding of a single vortex [6]. When the cylinder oscillates, this pattern may be modified depending on the frequency and amplitude of the forced oscillation. Patterns such as 2P, P+S, 2P+2S have been observed – here P means the shedding of a pair of vortices from the same side of the cylinder. These configurations are known as exotic wakes [5]. Bifurcation diagrams in the frequency-amplitude parameter plane (f, A) have been established, both experimentally and computationally, by many researchers, e.g. [6, 3].
The purpose of the present paper is to study the bifurcations which lead from one pattern to another. We primarily consider the case where the oscillation frequency of the cylinder is chosen as the Strouhal frequency, that is, the frequency of the vortex shedding for a fixed cylinder. For low values of the amplitude of the oscillating cylinder the wake is of type 2S, but for the dimensionless amplitude A around 1, the pattern changes into P+S. In the entire parameter range we consider the flow is periodic. The Reynolds number is 100, where it is reasonable to assume the flow is two-dimensional. Simulations are performed using the `oomph-lib` finite-element library [1].
In this study we characterize a flow configuration by the topology of the vorticity field, and define a vortex as a local extremum of vorticity. An extremal point of vorticity can be created together with a saddle point in a forward cusp bifurcation [2]. Similarly, an extremum and a saddle may merge and annihilate in a backward cusp bifurcation. We note that there are alternative definitions of a vortex, for instance the Q-criterion, which has also been of use in identifying creation and destruction of vortices [4].

Results

The results are summarized in Fig. 1. We focus on the negative vortices formed at the top of the cylinder. For the dimensionless amplitude A less than about 0.85, a positive and a negative vortex on each side of the cylinder are formed in the shear layers a few diameters downstream. Once created, the vortices persist and are advected downstream. This is a classical 2S wake. See Fig. 1(a) for a typical example. Increasing A to 1 as in Fig. 1(b), the filaments of vorticity (shear layers) emanating from the vortices may turn into individual vortices through a forward cusp bifurcation. These secondary vortices disappear again after a few cycles in a backward cusp bifurcation. This intermediary pattern cannot simply be described by the symbols P and S. Increasing A the secondary vortices persist for longer and longer times. For $A = 1.08$, Fig. 1(c), the secondary vortices never disappear again, and a full P+S wake is formed.
Hence, the transition process from a 2S to a P+S wake does not occur at a specific value of A, but is a sequence of bifurcations, possibly infinitely many, over a quite large interval of the parameter.

References

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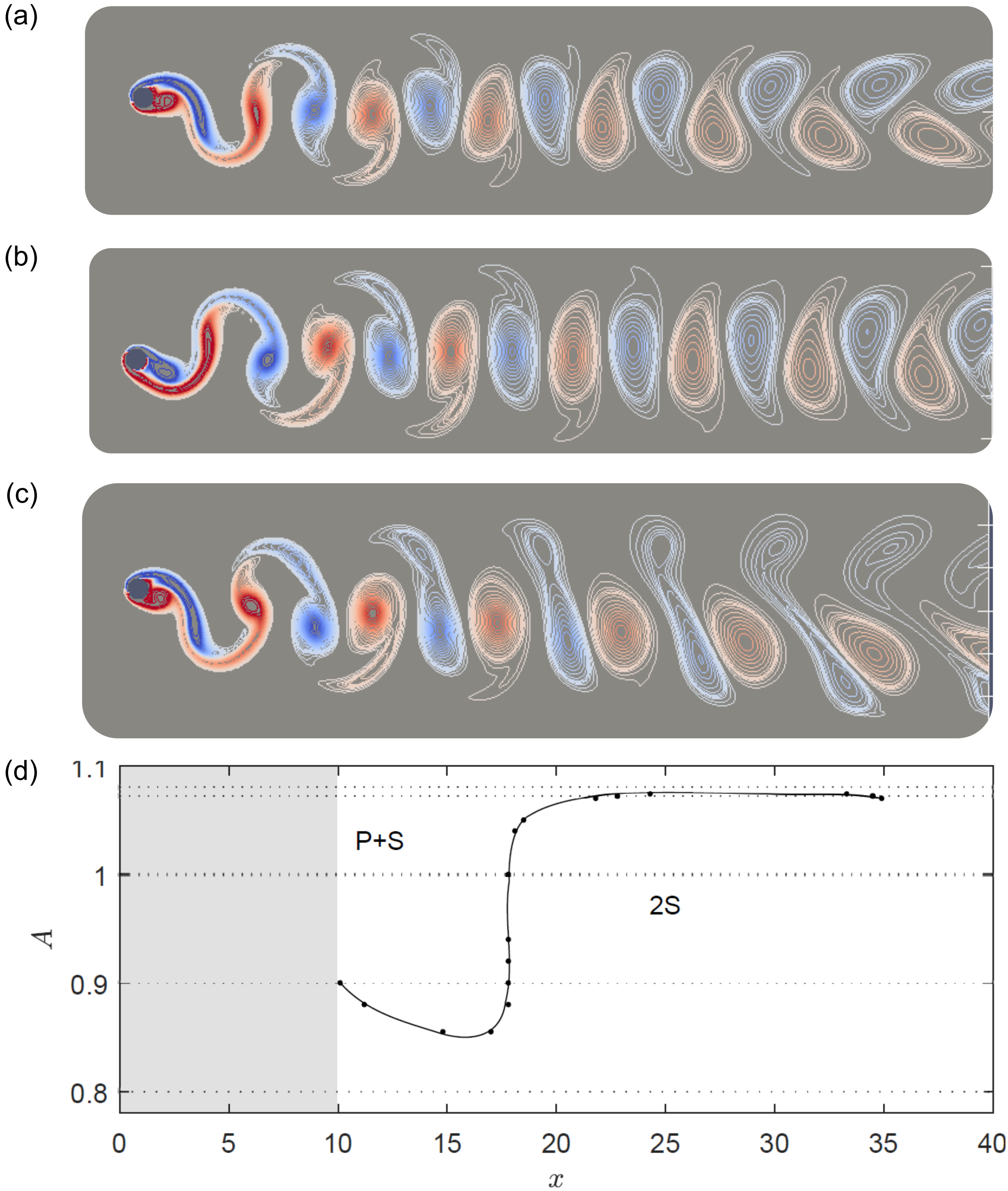

Figure 1: Snapshots of vorticity topologies for various values of the forcing amplitude A. (a): $A = 0.8$. (b): $A = 1$. (c): $A = 1.08$. (d): Bifurcation diagram showing the downstream distance x where forward or backward cusp bifurcations occur, at some time instant during the cycle. The shaded area shows the region where the shear layers are so thin that it is not possible to identify isolated extrema of vorticity.