SIR model for rumor propagation

Cristiane M. Batistela*, Manuel A. M. Cabrera*, Antonio C. B. Godoi* and José R. C. Piqueira*
* *Control Engineering Department, São Paulo University, São Paulo, Brazil*

Summary. Similarities exist between rumor propagation and disease spreading. In analogy with the SIR (Susceptible-Infected-Removed) epidemiological model, the ISS (Ignorant-Spreader-Stifler) rumor model has been developed. Here, the SIR model was modified with the new compartment, originating the ISSV (Ignorant-Spreader-Stifler-Verifier) rumor model. In this work, the model is presented and how its parameters are related to network characteristics is explained. This paper presents a system of differential equations that describes the spreading of a rumor in a new model, with a novel compartment, the verifier. By using concepts from Dynamical Systems Theory, equilibrium point is calculated, the stability and bifurcation conditions are derived. According parameters and initial conditions, the model is validated with numerical experiments. The relations among the model parameters in the several bifurcation conditions allow a rumor propagation design minimizing risks.

Introduction

With the rapid development of the Internet, the way to acquire information changed. There are a lot of platforms for this, as a typical example, social networks and they have been widely recognized. The media commonly releases their latest information through social networks, consequently can bring efficiency to daily life and information exchange, but it also results in gradual prevalence of online rumor.

Rumor, as a part of everyone is daily life, has often been defined as a type of social phenomenon with which some unconfirmed elaboration or annotation of the public interested events or issues spread on a large scale within a relatively short period of time through various channels, whether it is true or false [3].

People disseminate rumors as a mean of increasing awareness, orienting society, slandering others and so on. The spread of rumors can shape the public opinion and impact finantial markets and change population behavior [10].

The research on rumor propagation models started with development the stochastic rumor propagation proposed by Daley-Kendall, wich divided people into classes: ignorant, spreaders and stiflers, based on the infectious disease research method [1]. Maki-Thompson believed that rumors were disseminated through the two-way contact between the disseminators and other people in the crowd based on Markov chain [11]. Zanette first studied the dynamic behavior of rumor propagation in small world networks, and similarly to disease models he obtained the spreading threshold [17].

Based on the fact that social behavior and biological diseases are a result of interaction between individuals, many researchers have studied the dynamics of social systems by applying epidemiological models.

The epidemiological model proposed by Kermack and McKendrick, the SIR (Susceptible-Infected-Removed) compartmental model [7, 8, 9], has been used in several areas and modifications of this work allowed Daley and Kendall (DK) to propose a rumor spread model [1], the ISS (Ignorant-Spreader-Stifler) model wich shares commons characteristics with epidemic model [2], being important mark in rumor propagation.

The equations that model are similar with the ignorant and spreading populations from ISS model being analogous to susceptible and infected populations of the SIR model, respectively. The main difference between in the models, is that, in ISS, the stifler population plays a different role from the removed population because they does not propagation the rumor and remains in a constant state. In the other way, in SIR models, the removed individuals can be transformed into susceptible ones.

Based on the DK model, in which the participants were divided into three groups: one group of people who never heard the rumor (ignorant), one knowing and spreading the rumor (spreader), and one knowing the rumor but never spreading it (stifler). Afterwards, the rumor spreading model was refined with consideration of the forgetting mechanism [14] and by incorporating the effects of remembering mechanism in complex networks [18].

Various mathematical models for the propagation of a rumour within a population have been developed: the rumor spreading process with denial and skepticism [5], rumor spreading model with skepticism mechanism in social networks, epidemiological approach to model viral meme propagation [16], deterministic models for rumor transmission with constant and variable rumor in an age-independent population, models considering limited information exchange, which two types of rumors spread simultaneously among the crowd, stochastic rumor propagation model [6]. Recently, based on the classical SIR rumor propagation model, a new Susceptible-Infected-Hibernator-Removed model by adding a direct link from ignorants to stiflers and a new group hibernator [18].

Here, assuming an ISS model as a generalization of DK model and with analogies with the SIR model, the rumor spreading is studied, considering a new compartment, the verifier, ISSV (Ignorant-Spreader-Stifler-Verifier). The different dynamical propagation behaviors are possible, depending on how the nodes are connected. Furthermore, the model is studied under the assuming homogeneous distribution of social network that gives plausive qualitative results.

First, the differential equations representing the ISSV model are presented, followed by the stability analysis of the equilibrium points and the possibility of the bifurcation. Numerical experiments are performed to validade the analytical results. Finally, we present our conclusions and discuss some implications of the results.

ISSV Model: hypothesis and equations

The model proposed here is based on the original SIR model with analogies in epidemiologic models. A quantitative version of this model associated with compartmental model was analised by Moreno et al. [13].
By using concepts from Dynamics Systems Theory and assuming the ISS (Ignorant-Spreader-Stifler) models as generalization of DK model, the rumor spreading representation was studied, the several asymptotic behaviors are discussed and the possible bifurcations are shown by Piqueira [15] with the total population (T), divided into three groups: ignorants (I), spreaders (S) and stiflers (R).
Here, the proposed models have a new compartmant and each of the elements of the network can be one of the four different states: the first class are ignorants, and represents those individuals that never heard the rumor and are susceptible to be informed (I); the second group contains the individuals that are spreading the rumor, the spreaders (S); the third compartment, the verifier (V), can verify the rumor and finally, stiflers are those who know the rumor but that are no longer spreading it (R). This model is shown in Figure 1.

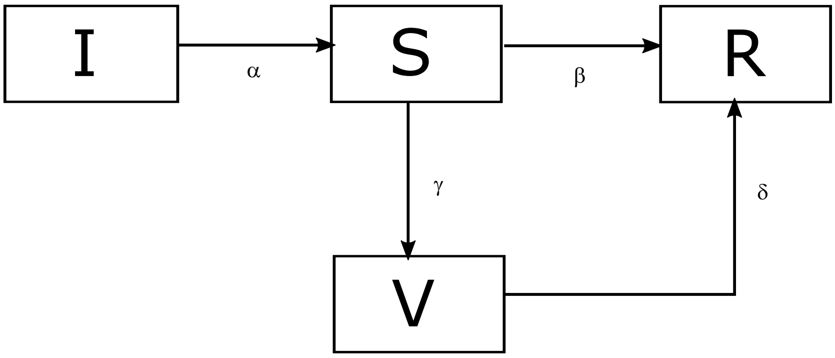

Figure 1: ISSV model.

The dynamical behavior of the spreading process depends on the way the individuals encounter each other. When a spreader meets an ignorant, the latter becomes a new spreader with probability α. Similar to the SIR model, the decay of the spreading process could be either when a spreader meets a stifler and contacts are supposed to have a probability equal to β or became verifier with the rate coefficient γ; in addition, verifier transform into stiflers with the rate coefficiente δ.
The dynamic equation for the populations I, S, V and R are:

$$\begin{cases} \dot{I} &= -\alpha IS; \\ \dot{S} &= \alpha IS - \gamma S - \beta S; \\ \dot{V} &= \gamma S - \delta V; \\ \dot{R} &= \beta S + \delta V. \end{cases} \tag{1}$$

The initial conditios are assumed to be $S(0) \geq 0$, $I(0) \geq 0$, $V(0) \geq 0$ and $R(0) \geq 0$. It is noticed that, for the model represented by (1), the total population $T = I + S + V + R$ remains constant and one of the equations can be expressed by a linear combination of the other three.

Bifurcation of the equilibrium states

In order to explore the influence of the verifiers during the rumor propagation, the dynamic system can be studied considering the equilibrium point of the model represented by (1).
The local stability of these points is analysed using Hartman-Grobman Theorem and the Jacobian derivative is calculated for each equilibrium points and the eigenvalues is calculated.
Examining the dynamical equation for the model, it is possible to determine that there is no spreader equilibrium point, i.e., equilibrium with spreaders nodes. This equilibrium states, spreader-free, corresponding to the absence the spreaders ($S = 0$) and the models in study presented one equilibrium condition, $P = (I, S, V, R) = (I^*, 0, 0, R^*)$.
To analyze the stability of this point, the general Jacobian (J), for the model (1), is constructed as:

$$J = \begin{bmatrix} -\alpha S & -\alpha I & 0 & 0 \\ \alpha S & \alpha I - \gamma - \beta & 0 & 0 \\ 0 & \gamma & -\delta & 0 \\ 0 & \beta & \delta & 0 \end{bmatrix}.$$

The Jacobian calculated in the equilibrium point P is reduced to:

$$J_P = \begin{bmatrix} 0 & -\alpha I & 0 & 0 \\ 0 & \alpha I - \gamma - \beta & 0 & 0 \\ 0 & \gamma & -\delta & 0 \\ 0 & \beta & \delta & 0 \end{bmatrix}.$$

Using MATLAB R2013a [12], the eigenvalues can be calculated: $\lambda_1 = 0$, $\lambda_2 = \alpha I - \gamma - \beta$, $\lambda_3 = -\delta$ and $\lambda_4 = 0$.
Considering the eigenvalues in J_P, for $\lambda_1 = 0$ and $\lambda_4 = 0$, one zero eigenvalue corresponds to the fact that the order of the dynamical system is three and other represent the central manifold [4] represented by the degenerated equilibrium. The eingenvalue $\lambda_2 = \alpha I - \gamma - \beta$, suggesting the detailed analyses because of the bifurcation possibility.
Examing the eigenvalue $\lambda_2 = \alpha I - \gamma - \beta$ and considering $T = I + R$ for the equilibrium point, the λ_2 can be rewritten as $\lambda_2 = \alpha T - \gamma - \beta$.
Analysing the signal of the second eigenvalue, the equilibrium point is asymptotically stable if $T < (\gamma + \beta)/\alpha$ and if $T > (\gamma + \beta)/\alpha$, the equilibrium point is unstable.

Numerical Experiments

To quantitative analyze the influence of verification nodes on the rumor network, numerical simulations of the model are performed considering the unitary total population ($T = I + S + V + R = 1$). Thus, the instantaneous values of the populations I, S, V, and R are expressed in percentage.
By using the Simulink tool from MATLAB R2013a [12], model simulations were performed to confirm the analytical results, showing the possible behaviors of the dynamical system when verifier compartments are present. The chosen parameters must be corrected to reproduce the results for different population sizes.
In order to simulate the stability of equilibrium point, the parameters must respect the condition $T < (\gamma + \beta)/\alpha$. Then, for $\alpha = 0.5$, β is chosen equal 0.3, $\gamma = 0.25$ and $\delta = 0.5$.
Starting with a condition with at least one percent spreader, the asymptotically stable equilibrium, P, is reachable and the rumor is not propagated. Figure 2 shows the time evolution of the model starting with $I_0 = 0.99$ and $S_0 = 0.01$. The spreaders become stiflers as well as a small percentage of ignorant. The resulting time evolution obtained and ending at the spreader-free equilibrium P.

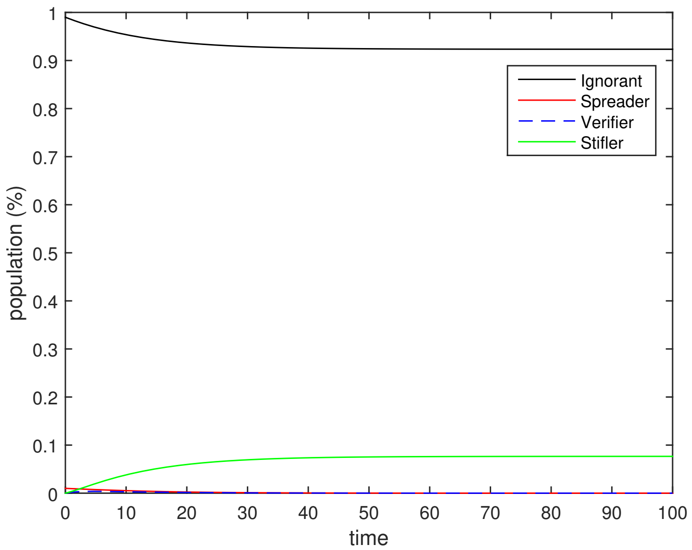

Figure 2: Simulating near stable equilibrium ($\alpha = 0.5$; $\beta = 0.3$; $\gamma = 0.25$; $\delta = 0.5$).

Remaining with the same parameter values and changing the initial conditions, the Figure 3 shows the spreader-free equilibrium P being reached for the initial conditions $I_0 = 0.89$, $S_0 = 0.10$ and $V_0 = 0.01$. In this situation, the spreaders and verifeirs become stiflers and some ignorants came to know the rumors.
Satisfying the condition $T < (\gamma + \beta)/\alpha$, the rumor is contained and decreasing the parameter α or increasing the parameter γ or β, rumor propagation dynamics are not changed.
In order to study the dynamics of the rumor propagation by varying the parameters, the initial condition is considered $I_0 = 0.99$ and $S_0 = 0.01$ and the β value was changed to 0.03. Then, for $\alpha = 0.5$, γ is chosen equal 0.25 and $\delta = 0.5$. Even decreasing the transformation rate of spreaders into stiffler, the rumor propogation occured by the increase of the stiflers. Figure 4 shows part of the ignorats become stifflers and in this situation $T > (\gamma + \beta)/\alpha$.

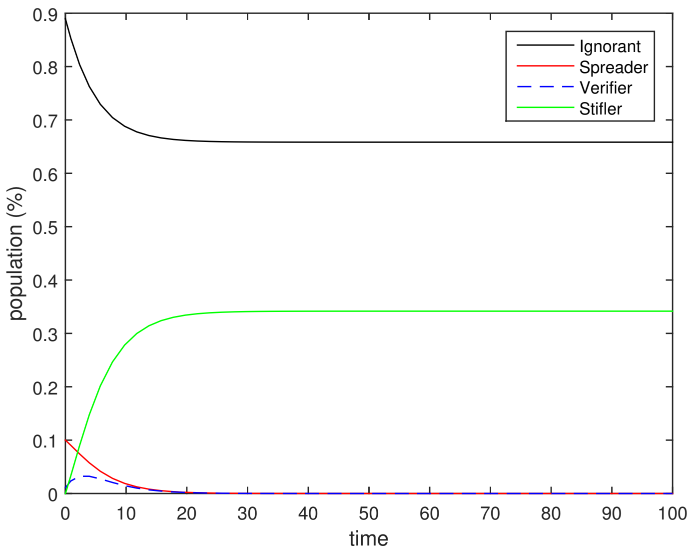

Figure 3: Simulating near stable equilibrium changing the initial percentage of spreader.

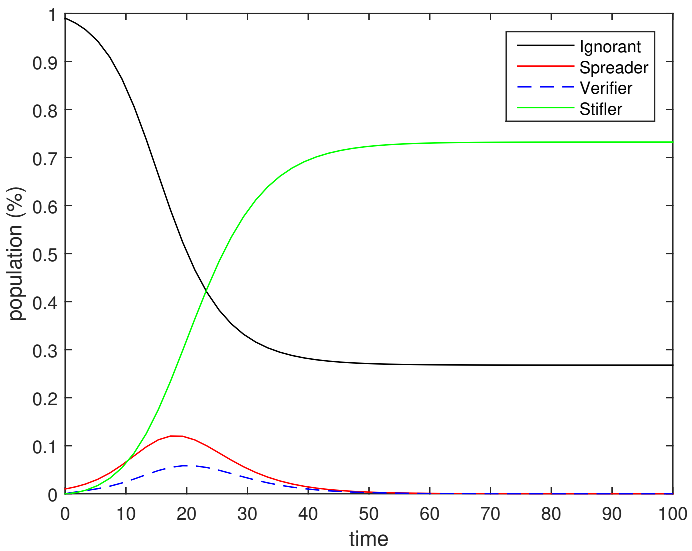

Figure 4: Changing β ($\alpha = 0.5$; $\beta = 0.03$; $\gamma = 0.25$; $\delta = 0.5$).

Changing the parameter α and considering the initial condition $I_0 = 0.98$, $S_0 = 0.01$ and $V_0 = 0.01$, Figure 5 shows the unstability of the equilibrium point P. The parameters are: $\alpha = 5$; $\beta = 0.03$; $\gamma = 0.25$; $\delta = 0.5$ and in this situation all the population become stiflers.
Considering $\delta = 0.05$ and the initial condition $I = 0.98$, $S = 0.01$ and $V = 0.01$, Figure 6 shows the influence of this parameter in the dynamics of the rumor propagation. The others parameters are: $\alpha = 5$; $\beta = 0.03$; $\gamma = 0.25$; $\delta = 0.05$ and in this situation all the population become stiflers. With the decreasing of this rate, the time evolution is changed but the qualitative behavior is the same.

Conclusions

The analysis of the ISSV model shows that, for a given total population, T, the main control parameters are probabilities α, β and γ measuring the efficient communication ignorant-spreader (α), spreader-stifler (β) and spreader-verifier (γ), respectively.
Perturbing near the equilibrium point with at least one spreader, for any combination the parameters, satisfying the condition $T > (\gamma + \beta)/\alpha$, the steady state is composed of a few ignorants, a lot of stiflers, and no spreaders, meaning that almost all the population heard the rumor.
Under the same initial conditions but satisfying the condition $T < (\gamma+\beta)/\alpha$, the equibrium point is not changed meaning that the rumor is not propagated.

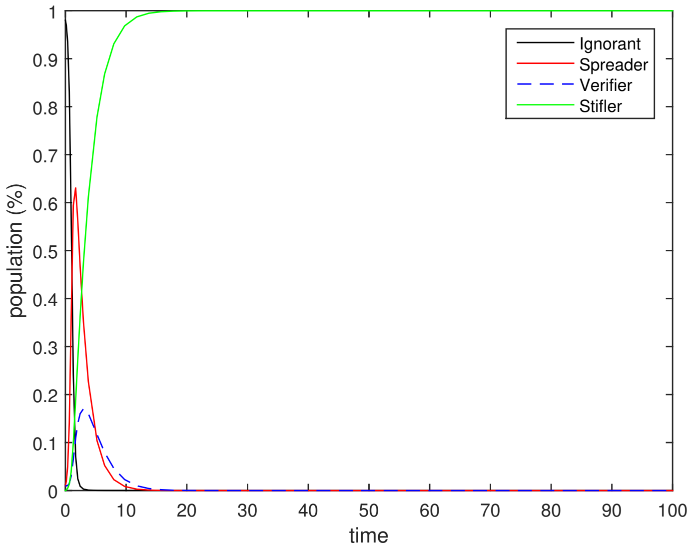

Figure 5: Changing α ($\alpha = 5$; $\beta = 0.03$; $\gamma = 0.25$; $\delta = 0.5$).

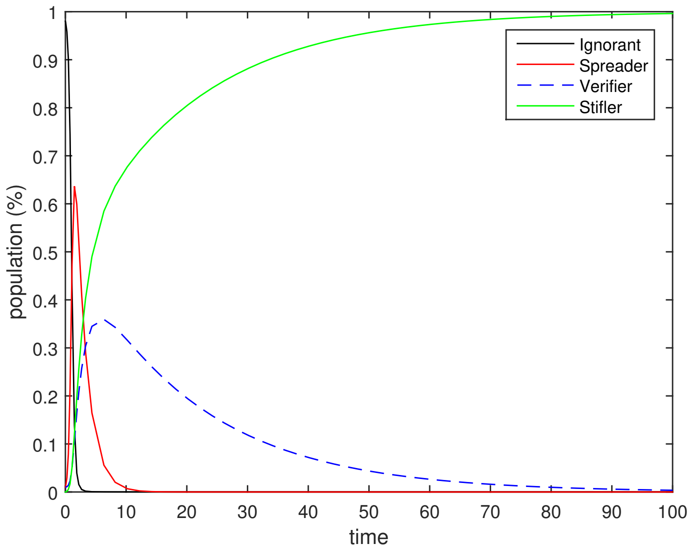

Figure 6: Changing α ($\alpha = 5$; $\beta = 0.03$; $\gamma = 0.25$; $\delta = 0.05$).

Changing the δ parameter, wich transform verifier into stifler, does not change the qualitative behavior associated with the spread of rumors.

References

[1] Daley, D. J., Kendall, D. G. (1964) Epidemics and rumours. *Nature* **204(4963)**: 1118-1118.

[2] Dietz, K. (1967) Epidemics and rumours: A survey. *Journal of the Royal Statistical Society: Series A (General)* **130(4)**: 505-528.

[3] Galam, S. (2003). Modelling rumors: the no plane Pentagon French hoax case. *Physica A: Statistical Mechanics and Its Applications* **320**: 571-580.

[4] Guckenheimer, J. Holmes, P. (1983) Nonlinear Oscillations, Dynamic Systems and Bifurcation of Vector Fields. Springer, NY.

[5] Huang, W. (2011) On rumour spreading with skepticism and denial. *Working paper*.

[6] Jia, F., Lv, G. (2018). Dynamic analysis of a stochastic rumor propagation model. *Physica A: Statistical Mechanics and its Applications* **490**: 613-623.

[7] Kermack, W. O., McKendrick, A. G. (1927) Contributions to the mathematical theory of epidemics. *Proceedings of the Royal Society Series A* **115(772)**: 700-721.

[8] Kermack, W. O., McKendrick, A. G. (1932) Contributions to the mathematical theory of epidemics. *Proceedings of the Royal Society Series A* **138(834)**: 55-83.

[9] Kermack, W. O., McKendrick, A. G. (1933) Contributions to the mathematical theory of epidemics. *Proceedings of the Royal Society Series A* **141(843)**: 94-122.

[10] Kimmel, A. J. (2004) Rumors and the financial marketplace. *The Journal of Behavioral Finance* **5(3)**: 134-141.

[11] Maki, D. P., Thompson, M. (1973). Mathematical models and applications: with emphasis on the social life, and management sciences. Prentice-Hall, Englewood Cliffs, NJ.

[12] Moler, C.B.(2004) Numerical Computing with MATLAB. SIAM, Philadelphia, PA.

[13] Moreno, Y., Nekovee, M., Pacheco, A. F. (2004) Dynamics of rumor spreading in complex networks.*Physical Review E* **69(6)**: 066130.

[14] Nekovee, M., Moreno, Y., Bianconi, G., Marsili, M. (2007) Theory of rumour spreading in complex social networks. *Physica A: Statistical Mechanics and its Applications* **374(1)**: 457-470.

[15] Piqueira, J. R. C. (2010) Rumor propagation model: An equilibrium study. *Mathematical Problems in Engineering* **2010**.

[16] Wang, L., Wood, B. C. (2011) An epidemiological approach to model the viral propagation of memes.*Applied Mathematical Modelling* **35(11)**: 5442-5447.

[17] Zanette, D. H. (2001). Critical behavior of propagation on small-world networks. *Physical Review E* **64(5)**: 050901.

[18] Zhao, L., Wang, J., Chen, Y., Wang, Q., Cheng, J., Cui, H. (2012). SIHR rumor spreading model in social networks. *Physica A: Statistical Mechanics and its Applications* **391(7)**: 2444-2453.