Analytical Criterion of Multimodal Snap-through Flutter of Thin-walled Panels

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Summary. Condition, under which thin-walled shallow panels can develop cyclical snap through dynamics due to airflow loads, is derived in the explicit analytical form. The methodology is based on the asymptotic of a perfectly flexible structure whose continuous manifold of equilibrium positions in the multidimensional space of configurations serves as a family of generating solutions. It is shown that supercritical airflows can cause global trajectories near such manifolds associated with a two-way snap through between the original and inverted positions of the panel.

Introduction

The term 'flutter' may cover quite different situations involving different mathematical tools of analyses [1]. The focus of the present study is flow induced dynamics with essentially nonlinear snap-through effects. Complexity of such type of problems is due to a multimodal strongly nonlinear structural behavior accompanied by a strong spatial coupling between the modes. The adapted elastic model of a shallow cylindrical panel ignores the longitudinal inertia term while taking into account the influence of membrane forces on the bending deformation as the only cause of geometrical nonlinearity [2, 3]. Assuming the presence of some initial imperfection and thus existence of multiple equilibrium positions, we define a snap-through flutter as the global panel dynamics with a cyclical self-sustained snap-through effect caused by the non-conservative aerodynamic load. Note that preserving the symmetric configuration during the snap-through motion would typically require a significant compression of the panel surface. As a result, any path through the least potential barrier must involve certain modal transitions avoiding significant tension-compression deformations that requires multimodal considerations. On one hand, this essentially complicates the analysis by increasing the problem dimension. However, on the other hand, increasing the dimension reveals a simple enough analytical estimate for the generating trajectory due to the asymptotic of a perfectly flexible panel. This represents a core of the approach, which assumes a global linearization near the manifold of a perfectly flexible panel [5, 6].

Technical details

Let us consider the elastic panel in a gas flow as schematically shown in Fig.1. The problem is reduced to the two-dimensional provided that the panel is subjected to a cylindrical bending. The panel thickness h is small compared to the amplitude of initial imperfection α, which itself is small compared to the span of the panel l. The outer surface of the panel interacts with the gas flow whose unperturbed velocity U is directed along the x-axis as shown in Fig.1.

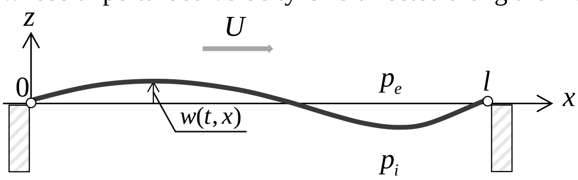


Figure 1: Aeroelastic model of a shallow panel in gas flow under the 'external' p_e and 'internal' p_i pressure loads.

The Lagrangian function of the panel is obtained based on the assumptions [2] as

$$L = \frac{1}{2}\rho h \int_0^l \left(\frac{\partial w}{\partial t}\right)^2 dx - V[w], \quad V[w] = \frac{Ehl}{2\left(1-\nu^2\right)} e[w]^2 + \frac{Eh^3}{24\left(1-\nu^2\right)} \int_0^l \left(-\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w_0}{\partial x^2}\right)^2 dx \tag{1}$$

$$e[w] = \frac{1}{2l}\int_0^l \left[\left(\frac{\partial w}{\partial x}\right)^2 - \left(\frac{\partial w_0}{\partial x}\right)^2\right] dx \tag{2}$$

where $e[w]$ is the longitudinal strain, $V[w]$ is the potential energy of elastic deformations, and $w_0 = w_0(x)$ is the shape of initial imperfection. Also, the variation of work done by the static pressure drop and nonconservative aerodynamic loads is given by

$$\delta A = \int_0^l f_z \delta w dx, \quad f_z(t,x) = \Delta p - 2\gamma\rho h \frac{\partial w}{\partial t} - \rho_\infty c U \frac{\partial w}{\partial x}, \quad \Delta p = p_i - p_e \tag{3}$$

where the aerodynamic load corresponds to a linearized equation of the so-called piston theory; see an overview in [4]. The manifold of zero-strain configurations is defined as

$$M_f = \{w : e[w] = 0\} \tag{4}$$

Then the linearization of the differential equation of motion near the manifold (4) is conducted with a continual version of transformation [5, 6] in the form

$$w = \tilde{w} + \varepsilon n \zeta, \quad n = \nabla_{\tilde{w}} e[\tilde{w}] / \left\|\nabla_{\tilde{w}} e[\tilde{w}]\right\|, \quad \varepsilon^2 = \left(h/\alpha\right)^2 / 12 \ll 1 \tag{5}$$

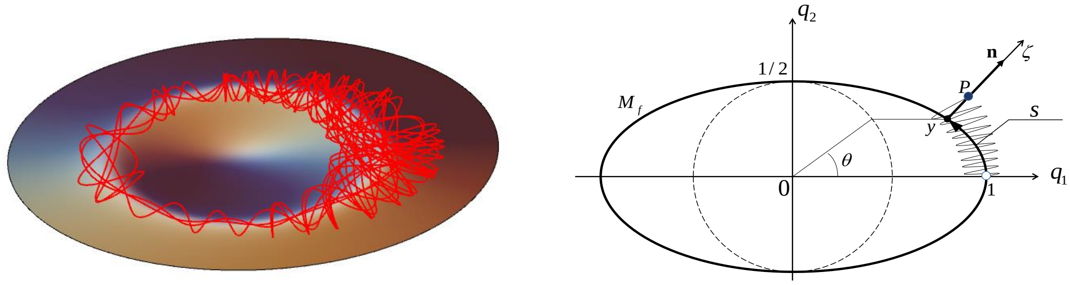
where, $\nabla_{\tilde{w}}$ is the so-called variational derivative, which is taken at the arbitrary "point" $\tilde{w} \in M_f$, and the symbol $\|\ldots\|$ denotes the Euclidian distance in the functional space of the panel configurations, therefore n (5) is a unit vector, which is always perpendicular to the manifold.

Two-mode illustration

Transformation (5) admits a clear visualization (Fig.2) after the following two-mode approximation of the panel' shape

$$w(t,x) = \alpha\left(q_1(t)\sin\frac{\pi x}{l} + q_2(t)\sin\frac{2\pi x}{l}\right), \quad w_0(x) = \alpha\sin\frac{\pi x}{l} \tag{6}$$

Note that expansion (6) provides an exact discretization of the corresponding free panel. However, it becomes just an approximation due to the presence of first derivative $\partial w/\partial x$ in the loading function.



(a) (b)

Figure 2: a) Two-mode snap-through trajectory around the potential hill of tension-compression deformations in the neighborhood of zero-strain manifold (4) M_f, and b) the local normal coordinate ζ near an arbitrary point $y(\theta)$ of the manifold M_f.

Fig. 2a shows a typical shape of the potential energy $V = V(q_1, q_2)$ with a sample trajectory of a free panel, which is averagely resembles the shape of elliptic zero-strain curve shown in Fig2b, including its generalized coordinate θ describing the tangential motion, and the coordinate ζ describing the fast normal component associated with tension-compression.

Result

Fig.2b justifies the averaging procedure with respect to the fast normal motion component compared to the slow tangential motion. This finally gives an asymptotic one-degree-of-freedom effectively conservative system for the tangential motion

$$\frac{d}{dt}\left[\frac{1+3\sin^2\theta}{4}\left(\frac{d\theta}{dt}\right)^2 + \varepsilon^2 V_{eff}(\theta,Q)\right] = O(\varepsilon^4), \quad V_{eff}(\theta,Q) = \frac{4}{3}Q\theta + (10+6\cos\theta)\sin^2\left(\frac{\theta}{2}\right) \tag{7}$$

Analyzing equation $\partial V_{eff}/\partial\theta = 0$ gives the critical number Q above which this equation has no real roots:

$$Q_* = \frac{2cl^3\rho_\infty U_*}{\pi^4 D} = \frac{1}{8}\sqrt{\frac{1}{2}\left(827 + 73\sqrt{73}\right)} \tag{8}$$

If $Q > Q_*$ then system (7) has no equilibrium points and as a result will continue to rotate around the ellipse.

Conclusion

It is shown that a strong enough airflow can result in global dynamic trajectories near such manifolds associated with two-way snap through between the original and inverted positions of the panel. The effective tangential to the manifold forces created by the airflow, dissipation, static pressure drop, and structural elasticity adequately determine conditions for qualitatively different dynamic regimes. In particular, it is shown that a strong enough airflow can result in global dynamic trajectories near such manifolds associated with a two-way snap through between the original and inverted positions of the panel. It must be noted that the developed approach is applicable to other problems with different types of loading as well as analyses of free vibrations accompanied by large amplitudes with or without snap through events.

References

[1] Dowell H. (2014) A Modern Course in Aeroelasticity: Solid Mechanics and Its Applications. Springer International Publishing.

[2] Kauderer H. (1958) *Nichtlineare Mechanik*. Berlin, Springer

[3] Bolotin V., Grishko A., Kounadis A., Gantes C., Roberts J. (1998) Influence of Initial Conditions on the Postcritical Behavior of a Nonlinear Aeroelastic System. *Nonlinear Dynamics* **15**, 63-81.

[4] Amabili M. (2008) *Nonlinear Vibrations and Stability of Shells and Plates.* Cambridge University Press.

[5] Pilipchuk V. (1986) Method of Investigating Nonlinear Dynamics Problems of Rectangular Plates with Initial Imperfections. *Soviet Appl. Mech.* **22**, 162-168.

[6] Nagaev R. F., Pilipchuk V.N. (1989) Nonlinear Dynamics of a Conservative System that Degenerates to a System with a Singular Set. *PMM* **53**(2), 190-195.