Equilibrium of a non-compressible cable subjected to unilateral constraints

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Summary. More realistic simulation of complex cable structures shall need the introduction of unilateral constraints to predict the global behavior of a whole structure without neglecting the influence of a realistic boundary condition. Indeed systems like ropeways (or belt drives) may exhibit behaviors that are strongly affected by friction at contacting zones between the cable (or belt material) and support (or pulley). This consideration also opens the gate to the study of the interaction between successive spans. Moreover the numerical simulation of these systems remains delicate due to their geometrical non linearity.

Introduction

Cables are the main features of many light engineering devices among which electric transmission lines, ropeways installation, cable car or lifts. The equilibria of cable system found their root in the sixteenth century. One the first author about the subject might be Leibniz with the two following works on the catenary curve [1, 2]. Far from this historical anecdote, today's engineering design still relies on those works or the parabolic approximation of weighting rope, first introduced by Galileo and made popular by Irvine [3]. From this monograph to current day, the richness of cable behaviours never ceased to be explored. Major studies in modelling and dynamics have been gathered by Rega in his review and work [4]. However, sophisticated boundary condition or constraints never had been deeply investigated for the cable which almost always perfectly hanged between two points and freely hanging. This is the aim of this work.

Impacts, collisions and friction play a key role in the transient regime of vibrating media by producing a high frequency content responses. Beyond the scope of the current application is the case of string music instrument which vibration against an obstacle explain the signal richness [5]. Similar phenomena are expected in a larger problem such as a ropeway and could explain the apparition of large oscillations when a cabin passes along the top of an intermediary pylon. Although cable dynamics have been studied intensively for the three last decades [3, 4, 6] and numerical challenges never ceased to be attacked [7, 8, 9, 10], there are still unanswered questions about different techniques of the Finite Element Method (FEM) [11, 12] applied to a cable which is subjected to nonsmooth constraints despite the existing bibliography about nonsmooth dynamics [13, 14].

Current focus is drawn on the equations for a cable which is subjected to the presence of a circular-shape obstacle and on a FEM associated with this problem. One may refer to the research work done in succession by Bruno et al. [15], Such et al. [16], Impollonia et al. [17] and Crussels-Girona et al. [10] about the equilibrium of a cable subjected to an intermediary pulley as mean of comparison.

System of interest

We are interested in the mechanics of a cable belonging to the Cartesian space $\mathbb{R}^3$ equipped with an orthogonal basis $(\underline{e}_x, \underline{e}_y, \underline{e}_z)$. A cable can be described as a curvilinear domain [6] which cannot resist any torque and compression due to its slenderness and its micro-structure. Each particle of this domain is associated to a unique curvilinear abscissa $S \in [0, L]$ where L stands for the reference length of the cable and a triplet of the Cartesian space:

$$q(S,t) = \begin{bmatrix} x(S,t) \\ y(S,t) \\ z(S,t) \end{bmatrix}. \tag{1}$$

The cable is subjected to external loads collected by $f(S,t) \in \mathbb{R}^2 \to \mathbb{R}^3$. The cable equilibrium is satisfied when tensile forces balance external solicitations and inertial force, which reads:

$$\rho \dot{v}(S,t) = \left(EA(\|q'(S,t)\| - 1)\frac{q'(S,t)}{\|q'(S,t)\|} \right)' + f(S,t) \tag{2}$$

Where $\dot{\bullet}$ and $\bullet'$ stand for derivatives of the variable $\bullet$ with respect to the time and space, respectively. The parameters ρ and EA are respectively the linear density of the cable and the cable rigidity. The velocity of the particle located at S is denoted by $v(S,t)$.

The equations of motion are supplemented by equality and inequality constraints to account for more detailed physics, such as the boundary conditions, the presence of obstacles, impacts or friction. The following constraints are introduced:

$$\text{(Equalities)} \quad a(q(S,t), S, t) = 0 \tag{3}$$

$$\text{(Inequalities)} \quad g(q(S,t), S, t) \geqslant 0 \tag{4}$$

The research of a solution for (2) is then performed in the admissible set $\mathbf{A}_{\text{dm}}$ given by:

$$\mathbf{A}_{\text{dm}} = \left\{ \begin{pmatrix} v(S,t) \\ q(S,t) \end{pmatrix},\ a\left(q(S,t), S, t\right) = 0,\ g\left(q(S,t), S, t\right) \geqslant 0,\ v \in \mathcal{RCBV}, q \in \mathcal{AC} \right\} \tag{5}$$

where the set of right continuous functions of bounded variations and the set of absolutely continuous functions are denoted by $\mathcal{RCBV}$ and $\mathcal{AC}$.

Finite element procedure

A finite element procedure is derived for the system given by (2)-(5). Following previous work [18], the unconstrained system is written:

$$\mathbf{M}\dot{\mathbf{v}}(t) + \mathbf{C}\mathbf{v}(t) + \mathbf{K}(\mathbf{q}(t))\mathbf{q}(t) - \mathbf{f}(t) = \mathbf{0}, \tag{6}$$

where $\mathbf{v}$ and $\mathbf{q}$ are finite-dimensional vectors collecting the unknowns at nodes. The equality and inequality constraints and their Jacobians are written at each nodes such that :

$$\mathbf{a}(\mathbf{q}, t) = \mathbf{0} \quad ; \quad \nabla^{\mathsf{T}}\mathbf{a}(t) = \mathbf{A}(t) \tag{7}$$
$$\mathbf{g}(\mathbf{q}, t) \geqslant \mathbf{0} \quad ; \quad \nabla^{\mathsf{T}}\mathbf{g}(t) = \mathbf{G}(t) \tag{8}$$

The mechanical system at stake can be formulated as a nonlinear complementarity system as follows:

$$\begin{cases} \mathbf{M}\dot{\mathbf{v}}(t) + \mathbf{C}\mathbf{v}(t) + \mathbf{K}(\mathbf{q}(t))\mathbf{q}(t) - \mathbf{f}(t) = \mathbf{A}(t)^{\mathsf{T}}\lambda + \mathbf{G}(t)^{\mathsf{T}}\mu \\ \dot{\mathbf{q}} = \mathbf{v} \\ \mathbf{a}(\mathbf{q}(t), t) = \mathbf{0} \\ \mathbf{0} \leqslant \mathbf{g}(\mathbf{q}(t), t) \perp \mu \geqslant \mathbf{0} \end{cases} \tag{9}$$

where λ and μ are Lagrange multipliers which can be physically interpreted as reaction forces provided that the Jacobians are normalized.
Let us introduce the following index sets:

$$\mathcal{E} = \{\beta, a^{\beta}(\mathbf{q}, t) = 0\}, \qquad \text{(index set of equality constraints)} \tag{10}$$
$$\mathcal{I} = \{\alpha, g^{\alpha}(\mathbf{q}, t) \geqslant 0\}, \qquad \text{(index set of inequality constraints)} \tag{11}$$
$$\tag{12}$$

and the following sets:

$$\mathcal{C}(t) = \{\mathbf{q}, \quad \mathbf{a}(\mathbf{q}, t) = \mathbf{0}, \quad \mathbf{g}(\mathbf{q}, t) \geqslant \mathbf{0}\} \tag{13}$$
$$\mathcal{N}_{\mathcal{C}}(\mathbf{q}) = \left\{ \mathbf{s} \in \mathbb{R}^n, \ \mathbf{s} = -\sum_{\beta \in \mathcal{E}} \lambda^{\beta}\mathbf{A}^{\beta} - \sum_{\alpha \in \mathcal{I}} \mu^{\alpha}\mathbf{G}^{\alpha}, \ \mu^{\alpha} \geqslant 0, \ \mu^{\alpha} g^{\alpha}(\mathbf{q}) = 0, \ \alpha \in \mathcal{I} \right\} \tag{14}$$

Time dependencies will be omitted in the following for conciseness. The dynamics given by (9) can be written as an inclusion into the normal cone $\mathcal{N}_{\mathcal{C}}$:

$$\begin{cases} \mathbf{r} = \mathbf{M}\dot{\mathbf{v}} + \mathbf{C}\mathbf{v} + \mathbf{K}(\mathbf{q})\mathbf{q} - \mathbf{f} \\ -\mathbf{r} \in \mathcal{N}_{\mathcal{C}}(\mathbf{q}) \\ \mathbf{q} = \mathbf{q}_0 + \displaystyle\int_0^t \mathbf{v}\ \mathrm{d}t \end{cases} \tag{15}$$

for the smooth phase of the motion. If the motion is nonsmooth, a differential measure equality must be written as:

$$\begin{cases} \mathbf{M}\ \mathrm{d}\mathbf{v} + [\mathbf{C}\mathbf{v} + \mathbf{K}(\mathbf{q})\mathbf{q} - \mathbf{f}]\ \mathrm{d}t = \ \mathrm{d}\mathbf{r} \\ \mathrm{d}\mathbf{q} = \mathbf{v}\ \mathrm{d}t \\ -\ \mathrm{d}\mathbf{r} \in \mathcal{N}_{\mathcal{C}}(\mathbf{q}) \end{cases} \tag{16}$$

where $\mathrm{d}t$ is the Lebesgue measure and $\mathrm{d}\mathbf{r}$ is a reaction force measure. See [14] for details about the decomposition of this measure. This equality means that the reaction force $\mathbf{r}$ tends to pull the system back into the admissible set $\mathcal{C}(t)$. An illustration for this situation is provided in Figure 1. Finally, to favor the use of the Moreau–Jean time–stepping scheme[19, 14], a second order Moreau sweeping process is formulated:

$$\begin{cases} \mathbf{M}\ \mathrm{d}\mathbf{v} + [\mathbf{C}\mathbf{v} + \mathbf{K}(\mathbf{q})\mathbf{q} - \mathbf{f}]\ \mathrm{d}t = \ \mathrm{d}\mathbf{r} \\ \mathrm{d}\mathbf{q} = \mathbf{v}\ \mathrm{d}t \\ -\ \mathrm{d}\mathbf{r} \in \mathcal{N}_{T_{\mathcal{C}}(\mathbf{q})}(\mathbf{v} + e\mathbf{v}^-), \end{cases} \tag{17}$$

where e is the coefficient of restitution and $T_{\mathcal{C}}$ the tangent cone to $\mathcal{C}$.
Numerical treatment of this system of equations will be at the heart of the following considerations.

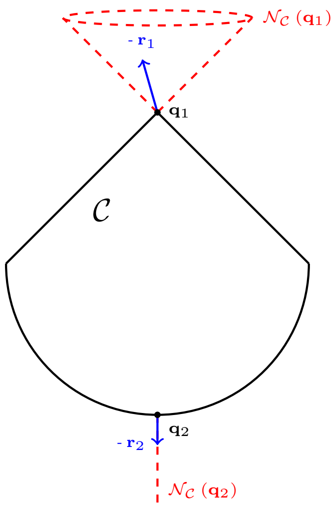

Figure 1: Normal cone and possible reaction forces in $\mathbf{q}_1$ and $\mathbf{q}_2$

Numerical treatment

In the Moreau–Jean time–stepping scheme [19], a θ-method is endowed to approximate integral quantities over a time-step h. For an arbitrary function ζ, it reads:

$$\int_{t_0}^{t_1} \zeta(t)\,\mathrm{d}t \approx h\left[\theta\zeta_1 + (1-\theta)\zeta_0\right] \tag{18}$$

where the subscript $\bullet_0$ (resp. $\bullet_1$) denotes the evaluation of $\bullet$ in t_0 (resp $t_1 = t_0 + h$). In case of the stiffness matrix $\mathbf{K}$, a linearized approximation is endowed as:

$$\mathbf{K}_1(\mathbf{q}_1) \approx \mathbf{K}_0(\mathbf{q}_0) + h\theta\nabla\mathbf{K}_0\mathbf{v}_1 + h(1-\theta)\nabla\mathbf{K}_0\mathbf{v}_0 \tag{19}$$

At last, the constraints are taken fully implicitly in the formulation such that:

$$\int_{t_0}^{t_1} \mathrm{d}\mathbf{r} \approx \mathbf{p}_1 + {\mathbf{A}_1}^{\intercal}\lambda_1 \tag{20}$$

where p_1 and λ_1 are homogeneous to impulses.
The equality and inequality constraints are considered at the velocity level and are linearized such that it reduces to:

$$\mathbf{A}_0^{\beta}\mathbf{v}_1^{\beta} = \mathbf{0}, \beta \in \mathcal{E} \tag{21}$$

$$\bar{g}_1^{\alpha} = g_0^{\alpha} + \frac{h}{2}\mathbf{G}_0^{\alpha}\mathbf{v}_0, \alpha \in \mathcal{I} \tag{22}$$

$$\begin{cases} \text{If } \bar{g}_1^{\alpha} \leqslant 0, 0 \leqslant \mu_1^{\alpha} \perp \mathbf{G}_0^{\alpha}(\mathbf{v}_1 + e\mathbf{v}_0) \geqslant 0 \\ \text{If } \bar{g}_1^{\alpha} > 0, \mu_1^{\alpha} = 0 \end{cases}$$

The satisfaction at previous time step is used to get the simplification in (21). Then the equality constraints are reduced to their first-order expand. The whole numerical scheme can be wrapped into the following system:

$$\begin{cases} \widehat{\mathbf{M}}\left(\mathbf{v}_1 - \mathbf{v}_0\right) - \widehat{\mathbf{f}} = \mathbf{p}_1 + {\mathbf{A}_0}^{\intercal}\lambda_1 \\ \mathbf{A}_0^{\beta}\mathbf{v}_1^{\beta} = \mathbf{0}, \beta \in \mathcal{E} \\ \bar{g}_1^{\alpha} \approx g_0^{\alpha} + \frac{h}{2}\mathbf{G}_0^{\alpha}\mathbf{v}_0, \alpha \in \mathcal{I} \\ \begin{cases} \text{If } \bar{g}_1^{\alpha} \leqslant 0, 0 \leqslant \mu_1^{\alpha} \perp \mathbf{G}_0^{\alpha}(\mathbf{v}_1 + e\mathbf{v}_0) \geqslant 0 \\ \text{If } \bar{g}_1^{\alpha} > 0, \mu_1^{\alpha} = 0 \end{cases} \\ \mathbf{q}_1 = \mathbf{q}_0 + h\theta\mathbf{v}_1 + h(1-\theta)\mathbf{v}_0 \end{cases}$$

where

$$\begin{aligned} \widehat{\mathbf{M}} &= \mathbf{M} + h\theta\mathbf{C} + h^2\theta^2\nabla\mathbf{K}_0 \\ \widehat{\mathbf{f}} &= h\theta\mathbf{f}_1 + h(1-\theta)\mathbf{f}_0 - h\mathbf{C}\mathbf{v}_0 \\ &\qquad - h\mathbf{K}_0\mathbf{q}_0 - h^2\theta\nabla\mathbf{K}_0\mathbf{v}_0. \\ \mathbf{p}_1 &= \sum_{\alpha}\mu_1^{\alpha}{\mathbf{G}_0^{\alpha}}^{\intercal} \end{aligned} \tag{23}$$

In order to obtain a mixed linear complementarity problem (MLCP) in a canonical form, the "free" velocity, $\mathbf{v}_f$ is computed for a given $\mathbf{v}_0$ solving the unconstrained system:

$$\mathbf{v}_f = \mathbf{v}_0 + \widehat{\mathbf{M}}^{-1}\widehat{\mathbf{f}} \tag{24}$$

The following iteration is referred as a one step nonsmooth problem (OSNSP) in the literature [14] which aims to obtain the correct impact estimation and the corrected velocity as the solution of:

$$\text{(OSNSP)} \begin{cases} \widehat{\mathbf{M}}\left(\mathbf{v}_1 - \mathbf{v}_f\right) = \mathbf{p}_1 + \mathbf{A}_0{}^{\intercal}\lambda_1 \\ \mathbf{A}_0\mathbf{v}_1 = \mathbf{0} \\ \mathbf{U}_1^\alpha = \mathbf{G}_0^\alpha \mathbf{v}_1 \\ \mathbf{p}_1^\alpha = \mu_1^\alpha \mathbf{G}_0^{\alpha\intercal} \\ \left.\begin{array}{l} \widehat{\mathbf{U}}_1^\alpha = \mathbf{U}_1^\alpha + e\mathbf{U}_0^\alpha \\ 0 \leqslant \mu_1^\alpha \perp \widehat{\mathbf{U}}_1^\alpha \geqslant 0 \end{array}\right\} \text{for } \alpha \in \mathcal{A}_{\textbf{ct}} \end{cases} \tag{25}$$

where the active set, $\mathcal{A}_{\textbf{ct}}$, is, in practice, a prediction of the index set of active constraints which will correspond to a violation of the unilateral constraints :

$$\mathcal{A}_{\textbf{ct}} = \{\alpha \in \mathcal{I}, \quad \bar{g}_1^\alpha \leqslant \mathbf{0}\}, \qquad \text{(index set of active inequality constraints)} \tag{26}$$

The problem (OSNSP) can be written in a canonical mixed linear complementarity problem (MLCP) as:

$$\begin{cases} \begin{bmatrix} \mathbf{A}_0\widehat{\mathbf{M}}^{-1}\mathbf{A}_0{}^{\intercal} & \mathbf{A}_0\widehat{\mathbf{M}}^{-1}\mathbf{G}_0{}^{\intercal} \\ \mathbf{G}_0\widehat{\mathbf{M}}^{-1}\mathbf{A}_0{}^{\intercal} & \mathbf{G}_0\widehat{\mathbf{M}}^{-1}\mathbf{G}_0{}^{\intercal} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \mu_1 \end{bmatrix} + \begin{bmatrix} \mathbf{A}_0\mathbf{v}_f \\ \mathbf{G}_0\mathbf{v}_f + e\,\mathbf{G}_0\mathbf{v}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \widehat{\mathbf{U}}_1 \end{bmatrix} \\ \mathbf{0} \leqslant \mu_1 \perp \widehat{\mathbf{U}}_1 \geqslant 0 \end{cases} \tag{27}$$

Constrained modes

Although the concept of modes has been thoroughly studied, the idea of a constrained mode is not clear yet. Looking at the mode as a preferential response for the system when solicited, we can extend the research for a periodic response to a system subjected to unilateral and bilateral constraints. Assuming the response will be small around a static equilibrium, the dynamics can be investigated in (9) via introducing a perturbation of the static equilibrium given by:

$$\text{(Static)} \begin{cases} \mathbf{K}(\mathbf{q})\mathbf{q} - \mathbf{f} = \mathbf{A}^{\intercal}\lambda + \sum\limits_{\alpha \in \mathcal{A}_{\textbf{ct}}} \mu^\alpha \mathbf{G}^{\alpha\intercal} \\ \mathbf{a}(\mathbf{q}) = \mathbf{0} \\ \mathbf{0} \leqslant \mathbf{g}(\mathbf{q}) \perp \mu \geqslant \mathbf{0} \end{cases} \tag{28}$$

Considering $\tilde{\mathbf{q}}(t) = \mathbf{q} + \mathbf{d}(t)$, $\tilde{\lambda} = \lambda + \lambda_{\mathbf{d}}$ and $\tilde{\mu} = \mu + \mu_{\mathbf{d}}$ in (9) and expansion around $\mathbf{q}$ combined with (28) yields:

$$\begin{cases} 0 = \mathbf{M}\ddot{\mathbf{d}} + \nabla[\mathbf{K}(\mathbf{q})\mathbf{q} - \mathbf{G}^{\intercal}\mu]\mathbf{d} - \mathbf{A}^{\intercal}\lambda_{\mathbf{d}} - \mathbf{G}^{\intercal}\mu_{\mathbf{d}} \\ 0 = \mathbf{A}\mathbf{d} \\ 0 = \mathbf{G}^\alpha \mathbf{d}^\alpha, \quad \alpha \in \mathcal{A}_{\textbf{ct}} \end{cases} \tag{29}$$

From the last two equations, suitable projections into the kernels of $\mathbf{A}$ and $\mathbf{G}$ should be done to obtain an eigenvalue problem. Let us consider $\mathbf{B}_1$ a basis of the kernel of $\mathbf{A}$ (then $\mathbf{B}_1{}^{\intercal}\mathbf{A}^{\intercal} = 0$) and $\mathbf{B}_2$ a basis of the kernel of $\mathbf{G}^{\intercal}\mathbf{B}_1$ (then $\mathbf{B}_2{}^{\intercal}\mathbf{G}^{\intercal}\mathbf{B}_1{}^{\intercal} = 0$). Let us refer to the product $\mathbf{B}_1\mathbf{B}_2$ as $\mathbf{B}$. One can manipulate (29) to obtain a classical evolution problem:

$$0 = \mathbf{B}^{\intercal}\mathbf{M}\mathbf{B}\ddot{\mathbf{d}} + \mathbf{B}^{\intercal}\nabla[\mathbf{K}(\mathbf{q})\mathbf{q} - \mathbf{G}^{\intercal}\mu]\mathbf{B}\mathbf{d} \tag{30}$$

which becomes a classic eigenvalue problem when the assumption of harmonic behavior is made for $\mathbf{d} = \exp(\mathrm{i}\omega t)\mathbf{d}_\omega$:

$$0 = \left(\widetilde{\mathbf{K}}(\mathbf{q}, \mu) - \omega^2\widetilde{\mathbf{M}}\right)\mathbf{d}_\omega \tag{31}$$

with $\mathrm{i}^2 = -1$. This methodology allows to trace linear vibrations around a steady-state. Main assumptions is that oscillations are "small" so that the active set of constraint remains unchanged as proposed by de Veubeke and Géradin [20].

Applications

Constrained mode veering

It is well known that the cable exhibits mode veering for some value of parameter [3, 21]. Here we are interested in the variation of modes for a suspended cable subjected to the presence of an intermediary obstacle of circular shape. That is to say we consider:

$$\mathbf{g}(\mathbf{q}^{\alpha})^{\alpha} = \|\mathbf{q}^{\alpha} - \mathbf{q}_R\| - R \tag{32}$$

where $\mathbf{q}_R$ and R stand for the circle center and its radius. The obstacle is translated along the $x-$axis as and the frequencies are computed for each equilibrium obtained as illustrated in Figure 4. We see that as one could expect in-plane (IP) modes or out-of-plane (OP) modes are obtained. Veering phenomenon occurs for some obstacle position, creating some possible scenarios of high resonances. We also see that the obtained modes are not single span modes and that an oscillation on the left of the obstacle can be accompanied with oscillation on its right, see Figure 2-3 which is opening the debate for a better physical description of multi-span cable systems.

Constrained dynamics

To account for the time stepping scheme (27) possible applications, we look at the dynamics of cable which only hanged at one end. The cable falls and may hit a vertical wall located at a given position, that is to say:

$$\mathbf{g}(\mathbf{q}) = \mathbf{B}(\mathbf{q} - \mathbf{q}_0) \tag{33}$$

where $\mathbf{B}$ is a diagonal matrix with 1 for arrays $(3j+1, 3j+1)$. Solving the MLCP (27) at each time step allows to compute the dynamics given by (9). Siconos is used to obtain the time response of the system depicted in Figure 5. A cable is falling from an initial catenary position and impacts a wall located on the edge of its initial support.

Conclusion

The global procedure to compute the dynamics of a nonlinear cable subjected to unilateral and bilateral constraints have been presented with several applications. Modes taking into account various types of constraints can be considered and provides interesting features of resonance and dynamical scenarios. The dynamics of an arbitrary system may be computed according to the nonsmooth contact dynamics and open the discussion about the influence of impact on the cable profile and tension.

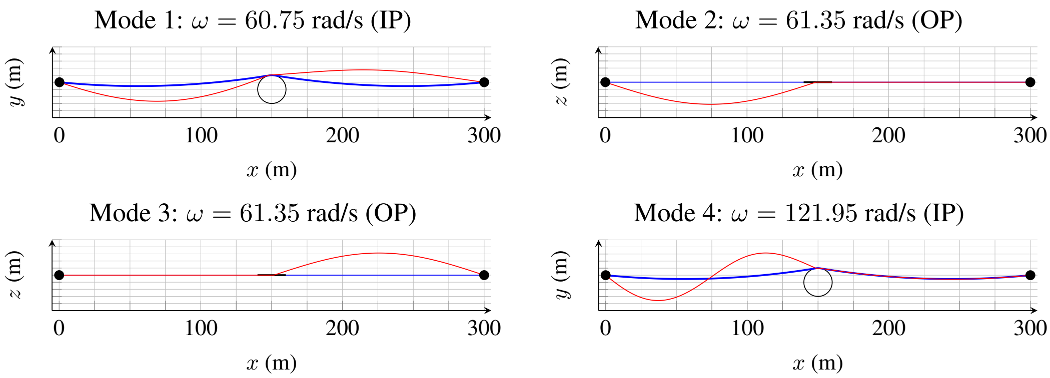

Figure 2: Mode-shapes obtained via FEM for a cable lying on a centred obstacle
Rest position (solid line blue) and perturbed position (solid line red)
L =301 m ; $\rho = 5.56$ kg/m ; $EA = 1.5$ GPa

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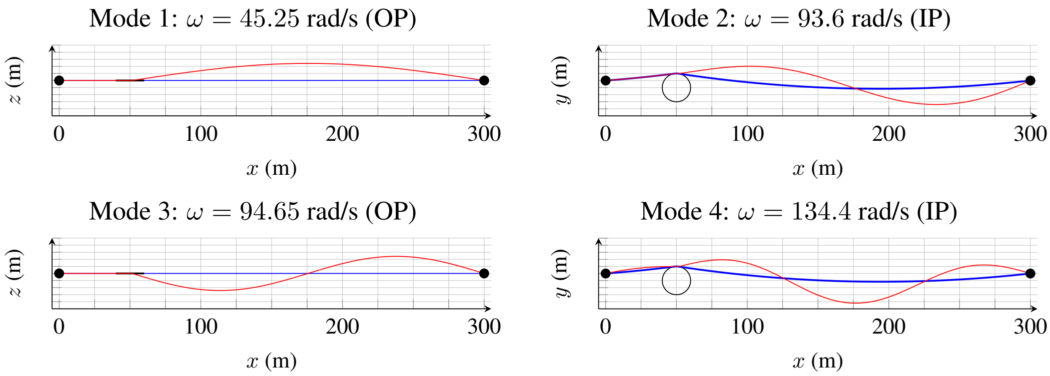

Figure 3: Mode-shapes obtained via FEM for a cable lying on a non-centred obstacle
Rest position (solid line ——) and perturbed position (solid line ——)
L =301 m ; $\rho = 5.56$ kg/m ; $EA = 1.5$ GPa

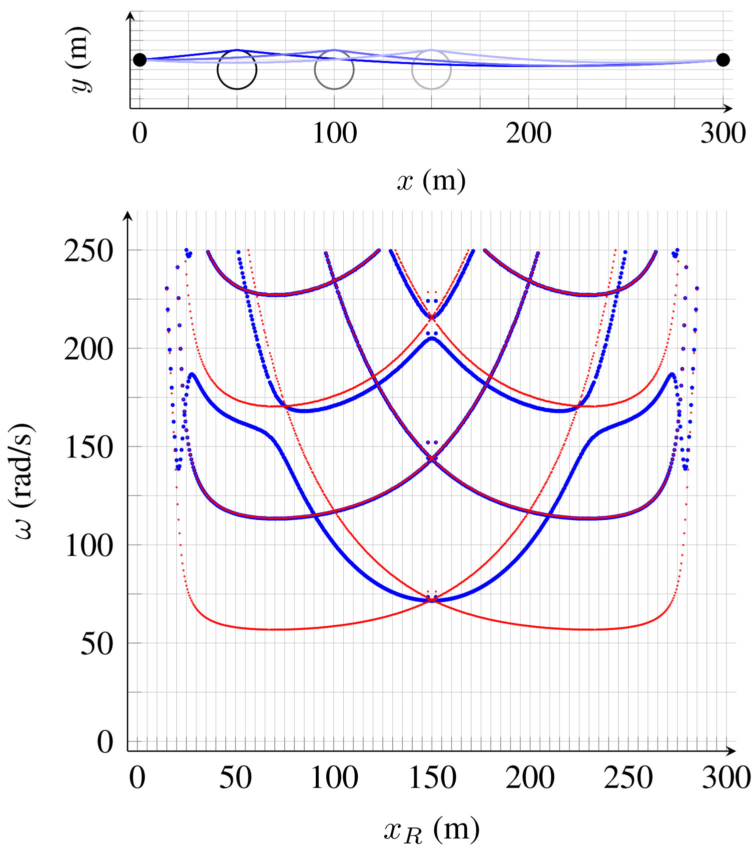

Figure 4: Evolution of the first frequencies for IP-modes (dotted line ·····) and the OP-modes (dotted line ·····) obtained numerically when the obstacle position is varying
L =300.6 m ; $\rho = 5.56$ kg/m ; $EA = 1.5$ GPa

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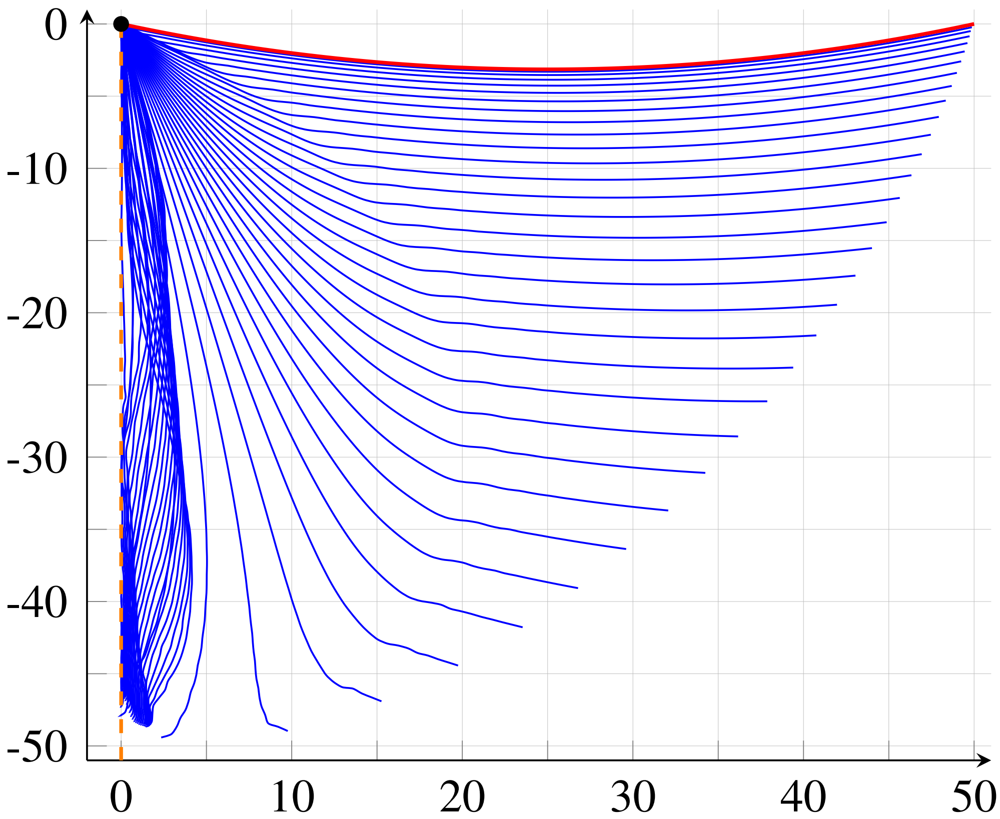

Figure 5: Station of a falling cable (solid line ——) impacting a wall (dashed line - - -) starting from resting catenary (solid line ——) L =50.6 m ; $\rho = 4$ kg/m ; $EA = 1.5$ GPa

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