# Reorientation of a rigid body by means of an auxiliary mass

Felix Chernousko*
*Institute for Problems in Mechanics, Russian Academy of Sciences, Moscow, Russia*

Summary. Possible control of the space orientation for a rigid body by means of an auxiliary movable point mass is considered. The motion of the point mass is proposed that provides the prescribed change of the body orientation.

## Introduction

Control of the space orientation of a rigid body can be implemented by means of an auxiliary internal mass that is equipped with an actuator and can move relative to the body. Two-dimensional motions of such systems in the absence of external forces are analyzed in [1, 2, 3] where time-optimal controls are obtained. Three-dimensional motions are considered in [4]. In the paper, a simple control is proposed which provides , in the absence of external forces, the prescribed change of the rigid body orientation by means of an internal movable mass.

## Basic equation

We consider a mechanical system consisting of a rigid body *P* of mass *M* and a particle *Q* of mass *m* (Fig. 1). Denote by *C* the center of mass of body *P* and by *O* the center of mass of system *P*+*Q*. Suppose that external forces are negligible and system *P*+*Q* is at rest at the initial time moment $t = 0$. Then its center of mass *O* is at rest for all *t*, whereas the momentum of system *P*+*Q* and its angular momentum stay constant and equal to zero. The following equation is derived from these conservation laws [1, 3, 4]:

$$\mathbf{J} \cdot \boldsymbol{\omega} + \mu M \mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}) = 0, \quad \mu = m/(M + m), \tag{1}$$

where **J** is the tensor of inertia of body *P* relative to its center of mass *C*, $\boldsymbol{\omega}$ is the angular velocity of body *P*, $\mathbf{r} = CQ$ is the position vector of point mass *Q* relative to *C*, and **v** is the velocity of point *Q* relative to body *P*.

## Reorientation

Let us introduce the Cartesian coordinate system $Cx_1x_2x_3$ connected with body *P*, its axes $Cx_i$ being principal central axes of inertia of body *P*, $i = 1, 2, 3$. Suppose that body *P* should be transferred from its initial state of rest to the prescribed terminal state of rest by means of an auxiliary particle *Q*. We assume that the initial and terminal positions of particle *Q* coincide with the center of mass *C* of body *P*. Hence, the required motion is the change of the orientation of body *P*.

This motion can be implemented by means of three successive plane turns of body *P* about its three principal central axes of inertia $Cx_i$, $i = 1, 2, 3$. For the rotation about axis $Cx_i$, body *P* must turn by a given angle $\Delta\varphi_i$ while the movable mass *Q* must start and finish its motion at point *C*. Both body *P* and particle *Q* should be at rest at the beginning and the end of this motion. Therefore, to design the required three-dimensional re-orientation, it is sufficient to construct such

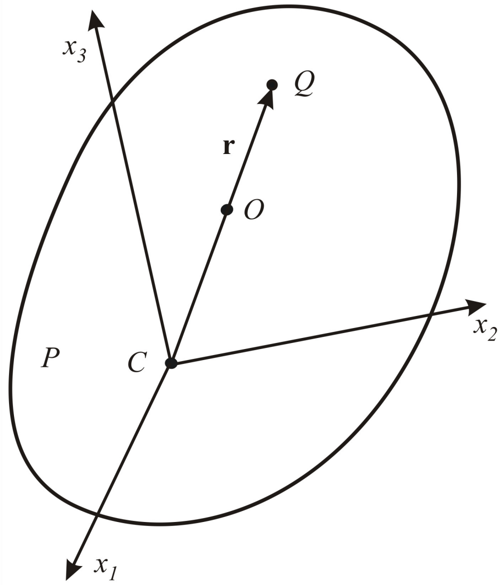


Figure 1: Mechanical system

plane motion of particle *Q*, with $\mathbf{r} = \mathbf{v} = 0$ at the beginning and the end of motion, that provides the required rotation of body *P*.

## Plane motion

Without loss of generality, let us consider rotation of body *P* about axis $Ox_3$. Point *Q* will move in plane $Cx_1x_2$, its trajectory begins and ends at point *C*. Time-optimal trajectories of point *Q* found in [1, 3] for small $\mu$, are circular arcs. Following this example, we will seek the trajectory of point *Q* as a circle with radius *R* passing through point *C*. Its center *S* can be chosen arbitrarily in plane $Cx_1x_2$.

Denote by *I* the moment of inertia of body *P* about axis $Cx_3$ and by $\varphi$ the angle of rotation of the body about this axis. Then the vectorial equation (1) is reduced to the following equation

$$\dot{\varphi} = -\frac{\mu M R^2(1-\cos\psi)}{I + 2\mu M R^2(1-\cos\psi)}\dot{\psi}, \tag{2}$$

where $\psi$ is the angle between radii *SC* and *SQ*; this angle defines the position of point *Q* along its circular trajectory. By integrating equation (2), we obtain

$$\varphi(t) = \frac{1}{a}\mathrm{Arctan}(a\tan\frac{\psi}{2}) - \frac{\psi}{2}, \tag{3}$$

where

$$a = (1 + 4\mu M R^2 I^{-1})^{1/2}. \tag{4}$$

According to equation (3), the angle $\varphi$ of rotation of body *P* about axis $Cx_3$ depends only on angle $\psi$, i.e., on the position of particle *Q* on its circular trajectory. When point mass *Q* makes the full rotation along its trajectory ($\psi = 2\pi$), body *P* turns by angle

$$\Delta\varphi = \pi(a^{-1} - 1), \quad -\pi < \Delta\varphi < 0.$$

To turn body *P* by an arbitrary angle, mass *Q* can make several (*n*) revolutions along its circular trajectory. Hence, the total angle of rotation $\Delta\varphi$ of body *P* can be estimated as follows

$$|\Delta\varphi| = \pi n(a-1)/a. \tag{5}$$

Using formulas (4) and (5), we obtain the expression for the radius *R* of the circular trajectory of point mass *Q*:

$$R = \frac{[I\alpha(1-\alpha/2)]^{1/2}}{(2\mu M)^{1/2}(1-\alpha)}, \quad \alpha = \frac{|\Delta\varphi|}{\pi n}. \tag{6}$$

To turn body *P* about axis $Cx_3$ by angle $\Delta\varphi$, particle *Q* should move in plane $Cx_1x_2$ along a circular trajectory of radius *R* given by (6). The center of the trajectory and the time history of motion $\psi(t)$ can be arbitrary.

This trajectory must pass through the center of mass *C* of body *P*, and the velocity of particle *Q* at point *C* must be zero. To decrease the domain of motion for particle *Q*, we can, according to equation (6), increase the number of revolutions *n*. The circular motions of particle *Q* relative to body *P* can be accomplished by means of rotating wheels.

## Conclusions

Possible motions of an auxiliary point mass relative to a rigid body are described which provide an arbitrary prescribed reorientation of the body in space in the absence of external forces.

**Acknowledgements**. The work is supported by Russian Science Foundation (Grant 18-11-00307).

## References


[1] Chernousko F.L. (2018) Optimal control of the motion of a two-mass system. *Doklady Mathematics* **97**:295-299.

[2] Shmatkov A.M. (2018) Time-optimal rotation of a body by displacement of a mass point. *Doklady Physics* **63**:337-341.

[3] Chernousko F.L. (2019) Optimal two-dimensional motions of a body controlled by a moving internal mass. *Multibody System Dynamics* **46**:381-398.

[4] Naumov N.Y., Chernousko F.L. (2019) Reorientation of a rigid body controlled by a movable internal mass. *Journal of Computer and Systems Sciences International* **58**(2):252-259.