Stable and Fast Identification of Continuous-Time Lur'e-Type Systems

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Summary. This abstract proposes an approach for parametric system identification for a class of continuous-time Lur'e-type systems. To overcome the computational drawbacks of numerical forward integration, the Mixed-Time-Frequency (MTF) algorithm is used to compute model responses in a fast way. These model responses are required to evaluate the cost function, which quantifies the mismatch between the measured and simulated steady-state output response. Furthermore, we show that the gradient of the cost function with respect to the model parameters can also be computed using the MTF algorithm. Hence, the MTF algorithm facilitates efficient use of global and local optimization methods to minimize the cost function, which yields the identified parameter set. Finally, by enforcing the identified model to be inside the set of convergent models, we certify a stability property of the identified model, which allows for safe generalization to other inputs than those used to train the model. The proposed approach is successfully applied in mechanical ventilation, where parameters of a first-principle model are identified. This case study highlights the benefits of the proposed approach.

Identification Problem

A practically relevant class of nonlinear systems is the class of Lur'e-type systems, see Figure 1. In such systems, the linear time-invariant (LTI) dynamics are captured in an LTI block and all the nonlinearities are captured in a static nonlinear block placed in the feedback loop. We consider the problem of parametric identification of so-called continuous-time *convergent* Lur'e-type systems. Convergent systems are systems that, for any bounded input, have a unique, globally asymptotically stable (GAS) steady-state solution that is bounded on the whole time axis [3]. For the class of Lur'e-type system, sufficient conditions for exponential convergence exist [5]. Our goal is to find parameters of the Lur'e type system that ensures the closest fit between the steady-state model response and the measured steady-state output response, while also ensuring that the identified model is convergent. We consider the case where both the input $w(t)$ and output $z(t)$ are scalar. The feedback signals $y(t)$ and $u(t)$ are considered *not measured*.
A property of convergent systems is that for T-periodic input $w(t)$, the steady-state output $z(t)$ is also T-periodic, facilitating the use of only steady-state data for the purpose of identification. The considered cost function, measuring the squared identification error, is given by

$$J(\theta) = \frac{1}{N}\sum_{k=0}^{N-1} \epsilon(t_k,\theta)^2 := \frac{1}{N}\sum_{k=0}^{N-1} \left(z_{simulated}(t_k,\theta) - z_{measured}(t_k)\right)^2 \tag{1}$$

with θ being the model parameter vector, parameterizing the LTI system matrices and the nonlinearity in Figure 1, N the number of samples in one period, t_k the sampling times (uniformly spaced) and $z_{simulated}(t,\theta)$, $z_{measured}(t)$, the simulated and measured steady-state response, respectively. Next, we define Θ as the set of parameters θ which renders the considered Lur'e-type model convergent. The objective is to minimize $J(\theta)$ in (1) while ensuring convergence by guaranteeing $\theta \in \Theta$. We consider the state dimension to be known and the model parametrization to be given by the user. The identification problem can now be formulated as follows:

$$\hat{\theta} = \arg\min_{\theta\in\Theta} J(\theta). \tag{2}$$

Cost Function Minimization

The constrained optimization problem (2) is solved in a two-step fashion. In the first step, initial parameter estimates are obtained. If the model is derived from first-principle modeling, then the user could provide initial parameter estimates based on physical insights. Otherwise, the best linear approximation [4] can be used, which yields a *linear* initial model in a fast way. Alternatively, any global parameter search algorithm [1] can be used, which results in a full *nonlinear* initial model, however, at the expense of computational time.
In the second step, a gradient-based search is used to optimize all parameters of the full nonlinear model. In order to evaluate the cost function (1), computation of the model response is required. Doing this using numerical forward

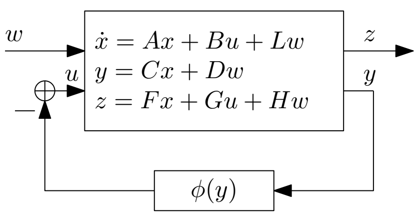

Figure 1: Considered Lur'e-type system.

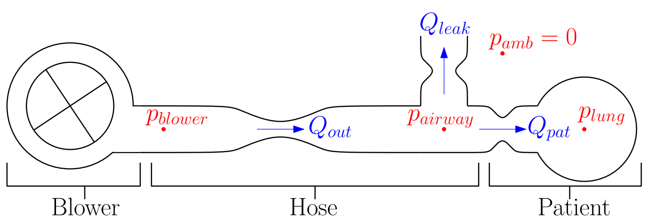

Figure 2: Experimental setup of mechanical ventilation.

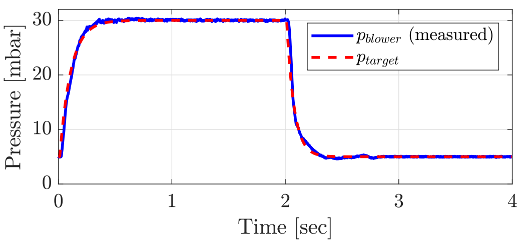

Figure 3: Realistic breathing pattern.

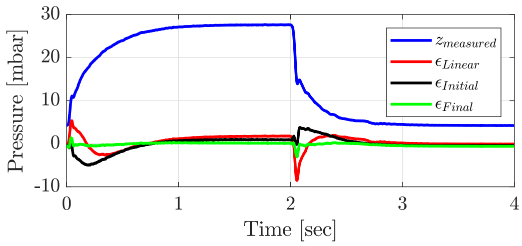

Figure 4: Measured response and remaining errors as defined in (1).

integration is a computationally expensive task. Therefore, for the class of convergent Lur'e-type systems, [3] developed the so-called Mixed-Time-Frequency (MTF) algorithm. This algorithm computes iteratively the response of the LTI block in frequency-domain and the response of the static nonlinearity in time-domain, which are both computationally efficient steps. It can be shown that for convergent Lur'e-type systems, this iterative computational approach is guaranteed to converge, ensuring the accurate and fast computation of the 'true' steady-state model response.

Besides the steady-state model response, also the gradient of the cost function (1) with respect to the model parameters is required in any gradient-based optimization approach to minimize the cost function in (1). One of our main contributions of this work is to show that this gradient can be obtained by simulation of a *parameter sensitivity system*, which is again a *convergent* Lur'e-type system. Hence, again the MTF algorithm can be used as a means of fast and accurate computation of the output response of this sensitivity system to obtain the gradient of the cost function. Using well-established optimization routines [2], the constrained optimization problem (2) can then be solved in a fast way by exploiting the computational benefits of the MTF algorithm.

Experimental Case Study in Mechanical Ventilation

The proposed approach is applied to find the parameters of a first-principles model of the mechanical ventilation setup schematically depicted in Figure 2. Mechanical ventilation is used in intensive care units to assist or stimulate respiration of patients who are unable to breathe on their own. The blower realizes the pressure p_{blower} by an internal control-loop which tracks the target breathing cycle p_{target}, both depicted in Figure 3. The measured pressure p_{blower} is considered as the input of the system. Air flows through a hose into the lungs of the patient, where at the patient-side of the hose the airway pressure p_{airway} is measured and considered as the output of the system. Also an intentional leakage component with known characteristics is present in order to refresh the air to the patient. Using first-principles modeling, a Lur'e-type model characterized by five parameters can be derived. The static nonlinearity in the model stems from the nonlinear pressure-flow characteristic of the hose, being characterized by a linear and quadratic resistance. Rather than using humans in these experiments, the ASL5000 breathing simulator is used, which simulates the lung behavior of patients, being characterized by a resistance and compliance parameter. The fifth parameter is the resistance of the leakage component, which is known by means of calibration. The case where the patient is fully sedated is considered, which implies no breathing activity from the patient.

A one minute experiment is performed where 15 periods of the 4 seconds periodic input depicted in Figure 3 are applied to the system. The average of the last 12 periods of p_{blower} and p_{airway} are used as *steady-state* input and output data, respectively, for the purpose of identification. Such a short experiment time is of crucial importance in this application as time is extremely costly in such medical settings. Parameters of an 'average' patient model are used to initialize a gradient-based exterior-point optimization algorithm to minimize the cost function in (1).

The measured steady-state output p_{airway} is depicted in Figure 4, together with the error obtained by the initial model $\epsilon_{Initial}$ (this is a model with *average* patient-hose parameters) and the model obtained after the gradient-based search ϵ_{Final}. For comparison, also the error of an identified *linear* model (using subspace techniques) is plotted in Figure 4. In this figure, the benefits of identifying a nonlinear model are clearly visible as it yields a much smaller error than the initial and linear model. This is also confirmed by the yielded cost (1), which is 2.55 for the initial model, 2.40 for the linear model and 0.17 for the final model. Furthermore, as we performed *parametric* system identification, the parameters of the model represent physical quantities that reveal important medical information on the patient and medical ventilation equipment, which are useful for medical personnel. To illustrate the computationally efficiency, a total of 1530 model responses were computed in *only* 13 seconds in the gradient-based search. The obtained model is guaranteed to exhibit the *convergence* property, which is highly instrumental for prediction purposes in controller design in mechanical ventilation.

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