Parametric excitation of an asynchronous Ziegler's column with a piezoelectric element

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Summary. The paper addresses the use of a piezoelectric element for energy-harvesting purposes in a 2 DOF Ziegler's column subjected to a harmonically varying follower force. Hence, two different destabilizing mechanisms are potentially present in this problem, namely the follower-force-driven flutter and the parametric instability. The system parameters are chosen in such a way that the column is tuned into the principal parametric resonance with respect to an asynchronous vibration mode. The initial voltage applied to the piezoelectric element is a key variable which either favours vibration control – as a result of the structural stiffening associated to the inverse effect, inherent to piezoelectric coupling – or energy harvesting. For brevity reasons, only small initial voltages will be considered herewith, focusing the case of enhanced vibrations to a stable post-critical steady state, which is suitable for energy harvesting. Due to the similarity of this problem to that of pipes carrying fluids or gases in harmonic flows, it is expected that similar behaviour could also appear there.

Introduction

The 2 DOF model of a column with a follower force P, forming an angle θ_2 with the vertical direction, as represented in Fig. 1, is the classic model studied in [1,2]. In this paper, it is further assumed that the follower force varies harmonically with time, so that $P(t) = P_0 + P_1 \cos \Omega t$. When $L_1 = L_2 = L$, $M_1 = 2M_2 = 2M$, $K_1 = K_2 = K$, $C_1 = C_2 = 0$ and also $\bar{p} = \frac{P_0 L}{K} = 2$, the system displays a stable asynchronous (in the sense it is *localised*) vibration mode $\mathbf{u}_1 = \{\theta_1 \quad \theta_2\}^T = \{0 \quad 1\}^T$ with natural frequency $\omega_{10} = \frac{1}{L}\sqrt{\frac{K}{M}}$. By the way, this value of $\bar{p} = 2$ is *slightly smaller* than the critical load $\bar{p}_{cr0} = \frac{7-2\sqrt{2}}{2}$ that causes instability of the trivial solution for the undamped model, as seen in [3]. At $\bar{p}_{cr0}$ a supercritical Hopf bifurcation takes place and a stable periodic attractor appears. It is worth noting that there is also a subcritical Hopf bifurcation at $\bar{p}_{cr0}$, indicating that if $\bar{p} < \bar{p}_{cr0}$, which happens to be the case, the basin of attraction of the stable trivial solution is very small due to the proximity to the critical load. It is further known that even an infinitesimal damping $C_1 = C_2 = \mu$ is capable of finitely reducing the critical load of the model to $\bar{p}_{cr\mu} = \frac{41}{28}$, as seen in [3]. Hence, the load $\bar{p} = 2$ that tunes the system into a *stable* asynchronous mode, which is *smaller* than the critical load for the undamped system, becomes *larger* than the critical one for the damped one.

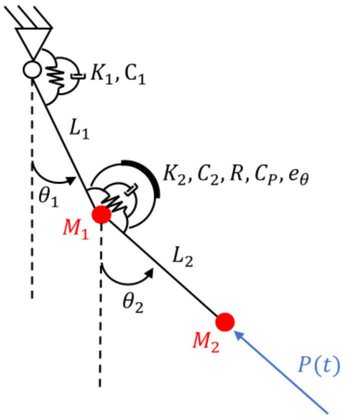
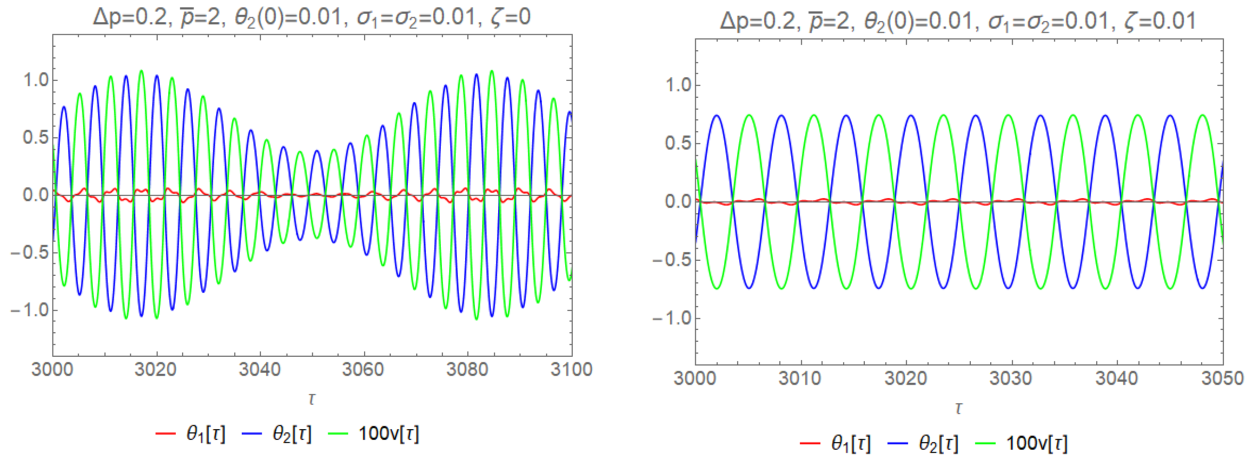


Figure 1: 2 DOF Ziegler column with follower force

A single piezoelectric element is inserted in parallel with spring K_2, with the following constitutive properties: electro-mechanical coupling term e_θ and capacitance C_P. Two dimensionless quantities associated with the piezoelectric parameters are introduced, namely $\sigma_1 = \frac{e_\theta^2}{C_P M L^2 \omega^2}$ and $\sigma_2 = \frac{1}{R C_P \omega}$, $\omega = \frac{1}{L}\sqrt{\frac{K}{M}}$ being the chosen reference frequency. The energy harvesting circuit is composed of an electrical resistance R. It is worth mentioning that if a new modal analysis is performed for the enlarged 3 DOF electro-mechanical model, in which the piezoelectric element voltage v is added as the third generalized coordinate, the previously found structural vibration mode $\mathbf{u}_1$ will not be practically affected, i.e. it will be *nearly-asynchronous*, provided σ_1 and σ_2 are *small*, which happens to be the case for $\sigma_1 = \sigma_2 = 0.01$. Yet, a slightly different natural frequency appears for the nearly-asynchronous mode, namely $\omega_1 = 1.02\frac{1}{L}\sqrt{\frac{K}{M}}$. Further, a free-body mode $\mathbf{u}_0 = \{\theta_1 \quad \theta_2 \quad v\}^T = \{-0.817 \quad -0.408 \quad 0.108\}^T$ will also appear, coinciding with the quasi-static deformed configuration due to an applied initial voltage to the piezoelectric element, known as the *inverse effect*.

Parametric excitation

In addition to the follower force scenario, already explained, parametric excitation is applied to the column by means of a harmonic variation of load $P(t) = P_0 + P_1 \cos \Omega t$, with $\Delta p = \frac{P_1 L}{K} = 0.20$ about $\bar{p} = \frac{P_0 L}{K} = 2$. The forcing frequency Ω is assumed to be twice the natural frequency $\omega_1 = 1.02 \frac{1}{L}\sqrt{\frac{K}{M}}$ of the nearly-asynchronous mode, so that the system is subjected to the *principal parametric instability* of the trivial configuration. Nevertheless, due to the kinematic and piezoelectric nonlinearities, it may be stabilised in a post-critical fluttering pattern. The piezoelectric element plays the role of an energy harvester. Fig. 2 illustrates the time histories for $\theta_1(t)$, $\theta_2(t)$ and $v(t)$ for a mechanically undamped ($\zeta = \frac{\mu}{2\sqrt{ML^2K}} = 0$) or a damped system ($\zeta = \frac{\mu}{2\sqrt{ML^2K}} = 0.01$). The assumed initial conditions were $\theta_1(0) = 0$, $\theta_2(0) = 0.01$ and $v(0) = 0$.



$\zeta = 0$ $\zeta = 0.01$

Figure 2: time histories for $\theta_1(t)$, $\theta_2(t)$ and $v(t)$

Although the follower-force-driven pattern alone *is not* characterised by localised vibrations, the parametric-instability pattern *does favour* localisation. It is seen that, for sufficiently large values of Δp, which happens to be the case here, the parametric instability pattern *prevails*, as seen in Fig.2, since $\theta_1(t) \approx 0$. The undamped case leads to a *post-critical amplitude-modulated nearly-asynchronous response*, whereas the damped one indicates a well-defined *stable periodic steady state nearly-asynchronous response*, in which $\theta_2(t)$ and $v(t)$ are in *anti-phase*.

Conclusions and final remarks

The two mechanical DOF Ziegler column model has been addressed in a scenario characterised by the coupling of four phenomena, namely: follower-force-driven flutter, parametric instability, modal asynchronicity and energy harvesting using a piezoelectric element. The paper is still the result of on-going research. The high system-parameter-space dimension didn't allow yet for an exhaustive study of all possible response regimes, such as those related to vibration control and the influence of the inverse-effect phenomenon provided by the piezoelectric constitutive properties. Future work will also look at the three mechanical DOF model of the Ziegler's column and the continuous model of the Beck's column. As already mentioned, the problem of pipes carrying fluids or gases in harmonic flows, due to common characteristics with the model studied herewith is of the author's interest.

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