Non-linear dynamic stability of mono-symmetric thin walled beams under random excitation

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Summary. The focus of my presentation is to study the non-linear dynamic stability of a viscoelastic thin walled beam with cross section having one axis of symmetry and subjected to a dynamical axial compressive load P(t) (generally eccentric).The equations of motion is derived and the method of stochastic averaging is used to decoupled the governing equations into Itô equations. For small damping and weak stochastic fluctuation, the expressions are derived for the moment Lyapunov exponent.

Introduction

Thin walled beams are widely used in construction, aircraft, ship building, etc. Also, with the increasing use of materials, there exists the need for analyzing viscoelastic structures under dynamic loading. Therefore, there are lots of works in both deterministic and stochastic domains. The general theory of thin walled beams was investigated by Vlasov [1] and the dynamic instability of elastic system was explained by Bolotin [2]. Stratonovich formulated the method of stochastic averaging [3] and it mathematically proved by Khasminskii [4]. There are the overwhelming number of papers in the literature. The modern theory of stochastic dynamic stability is founded on Lyapunov exponents and moment Lyapunov exponents [5]. Also for distinguishing among the different cases of resonances, the method of multiple scales is used.

Formulation

The flexural-torsional vibration of a thin walled beam with mono-symmetric I-section is presented in Fig.1. In the present article, the boundary condition is considered as both ends simply supported.

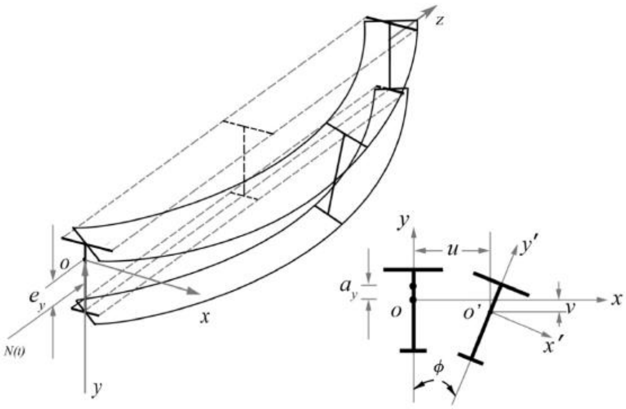

Figure 1

The equations of motion is defined as [1]. In these equations, we can see inertia components Δq, ΔM, ΔN which are nonlinearly related to the principal displacements u and ϕ. By locating a mass on the movable end [2] and adding the viscous damping and also replacing the elastic modulus by the Voltera operators E(1-H) [5], the governing partial differential equations become:

$$
\begin{aligned}
& E\,(1-\mathrm{H})\,J_y\left(\frac{\partial^4}{\partial z^4}\,u(z,t)\right)+P(t)\left(\frac{\partial^2}{\partial z^2}\,u(z,t)\right)+P(t)\,\left(a_y-e_y\right)\left(\frac{\partial^2}{\partial z^2}\,\phi(z,t)\right)+m\left(\frac{\partial^2}{\partial t^2}\,u(z,t)\right)\\
&\quad+\mathrm{D}_u\left(\frac{\partial}{\partial t}\,u(z,t)\right)+m\,a_y\left(\frac{\partial^2}{\partial t^2}\,\phi(z,t)\right)\\
&+\left(\frac{\partial^2}{\partial z^2}\,\Delta M(z,t)\right)\phi(z,t)+2\left(\frac{\partial}{\partial z}\,\Delta M(z,t)\right)\left(\frac{\partial}{\partial z}\,\phi(z,t)\right)+\Delta M(z,t)\left(\frac{\partial^2}{\partial z^2}\,\phi(z,t)\right)+\left(\frac{\partial}{\partial z}\,\Delta N(z,\right.\\
&\quad\left.t)\right)\left(\frac{\partial}{\partial z}\,u(z,t)+a_y\left(\frac{\partial}{\partial z}\,\phi(z,t)\right)\right)\\
&+\Delta N(z,t)\left(\frac{\partial^2}{\partial z^2}\,u(z,t)+a_y\left(\frac{\partial^2}{\partial z^2}\,\phi(z,t)\right)\right)=0
\end{aligned}
$$

(1)

$$
\begin{aligned}
&P(t)\left(a_y - e_y\right)\left(\frac{\partial^2}{\partial z^2}u(z,t)\right) + E(1-\mathrm{H})J_\omega\left(\frac{\partial^4}{\partial z^4}\phi(z,t)\right)\left[P(t)\left(r^2+2\beta_y e_y\right) - T(1-\mathrm{H})J_d\right]\left(\frac{\partial^2}{\partial z^2}\phi(z,\\
&t)\right) + m r^2\left(\frac{\partial^2}{\partial t^2}\phi(z,t)\right) + m a_y\left(\frac{\partial^2}{\partial t^2}u(z,t)\right) + \mathrm{D}_\phi\left(\frac{\partial}{\partial t}\phi(z,t)\right)\\
&+\left(\frac{\partial}{\partial z}\Delta N(z,t)\right)\left[r^2\left(\frac{\partial}{\partial z}\phi(z,t)\right) + a_y\left(\frac{\partial}{\partial z}u(z,t)\right)\right] + \Delta N(z,t)\left[r^2\left(\frac{\partial^2}{\partial z^2}\phi(z,t)\right) + a_y\left(\frac{\partial^2}{\partial z^2}u(z,\right.\right.\\
&\left.\left.t)\right)\right] + \Delta M(z,t)\left(\frac{\partial^2}{\partial z^2}u(z,t)\right) - 2\beta_y\left(\left(\frac{\partial}{\partial z}\Delta M(z,t)\right)\left(\frac{\partial}{\partial z}\phi(z,t)\right) + \Delta M(z,t)\left(\frac{\partial^2}{\partial z^2}\phi(z,t)\right)\right)\\
&-\Delta q(z,t)\, a_y\,\phi(z,t) = 0
\end{aligned}
\tag{2}
$$

With the aid of the Galerkin method, (1) and (2) can be converted into two ordinary differential equations. To show that damping, viscoelasticity and excitation are small, a parameter $0 < \varepsilon << 1$ is introduced. The axial thrust P(t) is assumed to be a stationary stochastic process.

Stochastic averaging

To apply the averaging method, one should consider first the unperturbed system which can be solved by the method of operators. The solutions are:

$$
u_1 = a_1\cos(\omega_1 t + \phi_1) + a_2\cos(\omega_2 t + \phi_2)\,,\; u_2 = \alpha_1 a_1\cos(\omega_1 t + \phi_1) + \alpha_2 a_2\cos(\omega_2 t + \phi_2) \tag{3}
$$

The method of variation of parameters is applied to determine the solutions of the perturbed system. Here, in the present article, because of the nonlinearities, there is a difference in the applying of this method with the other papers.

By averaging the responses over one period, we can expect to obtain accurate results. This may be done by applying the method of stochastic averaging. Through the method of stochastic averaging, the system can be approximated by Itô stochastic differential equations.

Moment Lyapunov exponents

To have a complete picture of dynamic stability, it is important to investigate both the moment and almost-sure stability which is the topic of the present work.
However, the stability of the pth moment of the solution of system is governed by the pth moment Lyapunov exponent defined by:

$$
\Lambda(p) = \lim_{t\to\infty}\frac{1}{t}\log E[\,\|\boldsymbol{u}\|^{\,p}] \tag{4}
$$

If the Λ is negative, the pth moment is asymptotically stable; otherwise, it is unstable [5]. One can obtain the largest Lyapunov exponent (5) through its relation with moment Lyapunov exponents (6).

$$
\lambda = \lim_{t\to\infty}\frac{1}{t}\log\|\boldsymbol{u}\| \tag{5}
$$

$$
\lambda_{\max} = \Lambda'(0) = \lim_{p\to 0}\frac{\Lambda(p)}{p} \tag{6}
$$

In this article, I use the moment Lyapunov exponents for analyzing the dynamic stability of system.

References

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