Self-sustained Vibrations and Dynamic Instability of Functionally Graded Carbon Nanotubes Reinforced Composite Shells

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Summary. Dynamic models of geometrical nonlinear deformations of functionally graded carbon nanotubes reinforced composite thin-walled structures are obtained. Reddy higher- order shear deformation theory is used to derive this model. The nonlinear system of high dimension ordinary differential equations, which describes the structure nonlinear vibrations, is obtained using the assumed-mode method. The linear piston theory is used to describe the supersonic flow. The losses of the cylindrical shell dynamic stability owing to the Hopf bifurcations are analyzed. The self- sustained vibrations, which describe the circumferential traveling waves flutter, occur due to this bifurcation. The harmonic balance method is applied to analyze these self-sustained vibrations. The properties of the circumferential traveling waves are analyzed. The dynamic instability of conical- cylindrical thin-walled nanocomposite structure is treated.

1. Introduction

Intensive researches in advanced materials, which have been widely used in aerospace engineering, are carried out in recent years. The extraordinary stiffness and tensile strength of carbon nanotubes (CNTs) make them well-suited as reinforcing components of composites. Computational approaches play a significant role in the development of the CNT reinforced composites by providing simulations results, which help to understand the behavior of nanocomposite structures. Effective elastic properties of CNTs are obtained in [1]. These properties are used to obtain effective elastic parameters of nanocomposites by the Mori-Tanaka methods. The effective mechanical properties of CNT reinforced composites are evaluated by 3D nanoscale representative volume element method by Liu, Chen [2]. The technique for developing constitutive models of CNTs reinforced polymer composite materials is proposed by Odegard and others [3].

The mechanical characteristics of the nanocomposites are analyzed experimentally by several researchers. The tensile tests of dog-bone shaped specimens were performed by Allaoui and others [4]. They obtained, that the Young's modulus and the yield strength have been doubled and quadrupled for composites with respectively 1 and 4 wt.% CNT in comparison with the pure resin matrix samples. Ci and Bai [5] systematically evaluate the stiffness of nanocomposite, when the CNTs reinforcement is used. The ultimate stresses experimental analysis of rubbery epoxy with CNTs reinforcement is treated by Richard and others [6].

The geometrical nonlinear vibrations of functionally graded CNTs reinforced composite cylindrical shell is analyzed by using the higher- order shear deformation theory. The self- sustained vibrations of the cylindrical shell interacted with the supersonic flow are analyzed numerically. The piston theory is applied to describe the supersonic flow.

The assumed-mode method is used to analyze the nonlinear vibrations of functionally graded CNTs reinforced composite cylindrical shell. The high dimension nonlinear system of the ordinary differential equations is obtained. The harmonic balance method with monoharmonic approximation of the self-sustained vibrations is applied to analyze the self-sustained vibrations. The system of the nonlinear algebraic equations with respect to the self-sustained vibrations amplitudes is derived.

The dynamic instability of the shell with different types of composite CNTs reinforcement is analyzed. As follows from the numerical analysis, the type of composite reinforcement affects essentially on the system parameters, when the flutter occurs. If the type of composite reinforcement is changed, the system parameters of the flutter origination can be changed twice.

The effect of the composite reinforcement type on the self-sustained vibrations is investigated. As follows from the data of the numerical simulations, the CNTs reinforcements of the composite shell affect essentially on the amplitudes of the self-sustained vibrations.

2. Problem formulation and main equations

The cylindrical shell with constant thickness h in supersonic flow is treated. The dynamic stress-strain state of the cylindrical shell is analyzed in curvilinear coordinate system (x, θ, z). Three projections of the middle surface displacements and rotations of the middle surface normal are chosen as the main unknowns. The dynamic instability of the cylindrical shell in supersonic flow is analyzed. This instability results in an increase of the vibrations amplitudes. In this case, the shell geometrical nonlinear deformations occur. This growth of the amplitudes is limited due to the shell geometrical nonlinear behavior. Then three projections of the middle surface displacements u, v, w are moderate and shell strains are small. The Hooke's law is true.

The material of the shell is functionally graded CNTs reinforced composite. CNTs are assumed to be uniaxially aligned. Five types of CNTs reinforced are considered. UD denotes uniform CNTs reinforced in the transverse direction of the cylindrical shell. The rest types of CNTs reinforced are FGV, FG**Λ**, FGX and FGO. They have variable CNTs dispersion. The shell material is functionally graded. The part of the volume for uniform distribution, which is occupied by CNTs, is denoted by V_{CNT}^{*}.

As the shell material is functionally graded and composite, shear is taken into account. The shear modulus takes the form: $G_{13}(z) = G_{12}(z)$; $G_{23}(z) = G_{12}(z)$. The Hooke's law is the following:

$$\begin{bmatrix}\sigma_{XX}\\ \sigma_{\theta\theta}\end{bmatrix} = \begin{bmatrix}Q_{11}(z) & Q_{12}(z)\\ Q_{12}(z) & Q_{22}(z)\end{bmatrix}\begin{bmatrix}\varepsilon_{XX}\\ \varepsilon_{\theta\theta}\end{bmatrix}; \tag{1}$$

$$\sigma_{\theta Z} = G_{23}(z)\gamma_{\theta Z};$$

$$\sigma_{XZ} = G_{13}(z)\gamma_{XZ};$$

$$\sigma_{X\theta} = G_{12}(z)\gamma_{X\theta}.$$

The projections of the shell points displacements, which are placed on the z distance from the middle surface, are denoted by $u_x(x,\theta,t,z), u_\theta(x,\theta,t,z)$ and $u_z(x,\theta,t,z)$. The higher- order shear deformation theory is used to describe the shell displacements:

$$u_x(x,\theta,t,z) = u(x,\theta,t) + z\phi_1(x,\theta,t) + z^2\psi_1(x,\theta,t) + z^3\gamma_1(x,\theta,t); \tag{2}$$

$$u_\theta(x,\theta,t,z) = \left(1+\frac{z}{R}\right)v(x,\theta,t) + z\phi_2(x,\theta,t) + z^2\psi_2(x,\theta,t) + z^3\gamma_2(x,\theta,t);$$

$$u_z(x,\theta,t,z) = w(x,\theta,t),$$

where R is radius of the cylindrical shell; ϕ_1 and ϕ_1 are the rotations of the middle surface normal about the θ and x axes, respectively.

3. Results and discussions

As a result of the numerical analysis, the bifurcation diagram is calculated (Fig.1). The self-sustained vibrations are originated as a result of the Hopf bifurcation. The influence of the types of CNTs reinforced is analyzed.

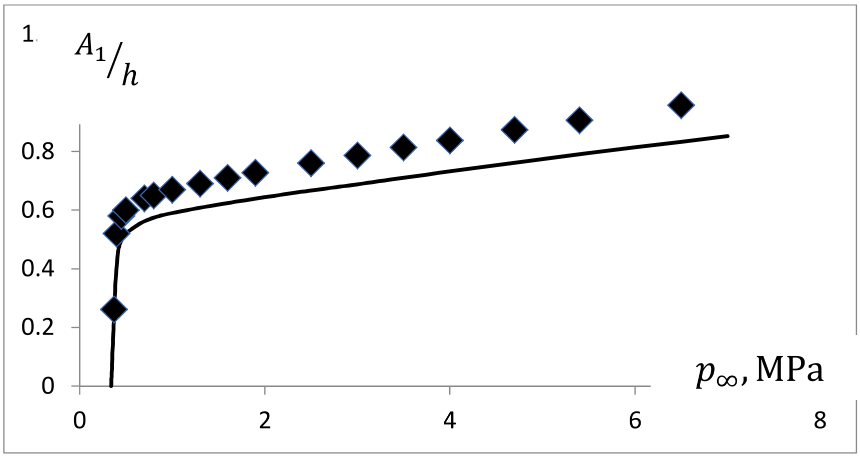

Figure 1: The response diagram of the self-sustained vibrations. The amplitudes $A_1 h^{-1}$ versus the free stream static pressure p_∞ is shown

This study was funded by National Research Foundation of Ukraine (grant number 128/02.2020) and by National Academy of Science of Ukraine (grant 0120U101241).

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