New simple oscillator model describing ice-induced vibrations of an offshore structure

A. K. Abramian*, S. A. Vakulenko†
**Institute for Problems in Mechanical Engineering RAS, Saint Petersburg, Russia*
†*Institute for Problems in Mechanical Engineering RAS, Saint Petersburg, Russia*

Summary. In this paper a new simple oscillator model is considered describing ice induced vibrations of upstanding, water surrounded, and bottom-founded offshore structures. Existing models are extended by taking into account deformations of an ice floe, and a moving contact interaction between an ice rod, which is cut out from the floe, and the oscillator which represents the offshore structure. Special attention is paid to a type of ice-induced vibrations (IIV) of structures, known as frequency lock-in, and characterized by having the dominant frequency of the ice forces near a natural frequency of the structure. The instability onset,induced by resonance for the oscillator and generated by the ice rod structure interaction, is studied in detail.

Statement of the problem

Vertically-sided, bottom-founded offshore structures occasionally experience sustained vibrations due to drifting ice sheets crushing against them. Usually, three regimes of interaction are distinguished: intermittent crushing, frequency lock in, and continuous brittle crushing. In this paper, we introduce a mathematical model for a special type of ice- induced vibrations (IIV) of structures, known as frequency lock-in, and characterized by having the dominant frequency of the ice forces near a natural frequency of the structure. In this paper, we propose a model extending the previous ones, in particular, those suggested in [1, 2, 3, 4]. A novelty with respect to the previous investigations is that we study the ice deformations in more detail including nonlinear deformation. We describe deformations of the ice rod taking into account possible viscous ice behaviour, and a moving contact between the structure and the ice. Following [1, 2, 3, 4] we consider an offshore structure as a one-degree-of- freedom oscillator and the ice floe as a system of ice rods, whose properties include local failure. In particular, we consider a simple oscillator, which interacts with one of such rods. Considering only one rod we take into account the connections of this rod with the others following the approach suggested in [5]. In order to simulate the behavior of the structure during the frequency lock-in regime the condition was set that the ice should always be in contact with the structure. On the basis of this requirement the equation describing the simple oscillator dynamics is given by:

$$q_{tt} + \Omega^2 q + \alpha q_t = \mu, \tag{1}$$

where $q = q(t)$ is the oscillator displacement, $\Omega^2 = \frac{G}{M}$, where Ω is the oscillator frequency, and M and G are the mass and rigidity of the oscillator respectively, and $\alpha > 0$ is a positive damping coefficient. For (1) the following initial conditions are used:

$$q(0) = 0, \quad q_t(0) = 0. \tag{2}$$

The term μ in (1) defines a force which is acting on the oscillator due to the ice rod, and has the form

$$\mu = \frac{EF}{M}(u_x + \frac{\delta_0}{E} u_{xt} + u_x^2)|_{x=q(t)}, \tag{3}$$

where $u = u(x,t)$ is the longitudinal ice rod displacement, E is an ice Young's modulus, F is the ice rod cross sectional area, and δ_0 is the ice internal, structural damping coefficient. The term u_x in the right hand side of (3) defines the contribution of linear deformations, and the term u_{xt} is the ice viscosity. The following equation describes the dynamics of the ice rod, which is defined on the semi-infinite domain $I_q = \{x : q < x < \infty\}$.

$$r u_{xx} - m u_{tt} + \delta_0 u_{xxt} = Q, \tag{4}$$

$$Q = -\beta(s_t - u_t) - k_0(s - u), \tag{5}$$

where $u(x,t)$ is the unknown ice rod displacement, m is the ice rod mass per unit length, Q is a force occurring in the rod due to its side-surface contact with others ice rods in the space around the rod in the ice floe that is considered in (4). The ice floe is drifting along the x-axis. The parameters β,r,δ_0, and k_0 are positive. The coefficient β is the ice friction coefficient during its side-surface contact, the coefficient r is the coefficient relating the shearing stress and the strain in the ice floe, and thus defines "the load spreading capacity of the foundation" according to [5]. The parameter k_0 characterizes the rod compression which is caused by stresses due to the ice rod compression in the transverse direction by other ice rods. In other words, the ice floe behavior can be modelled by a generalized spring and a generalized dashpot. The function $s(t)$ describes the shift of the ice rod, and $s(t)$ is defined by $s(t) = -vt + \rho(t)$, where $v > 0$ is the relative ice velocity, and $\rho(t) = \sum_{n=1}^{\infty} d_n H(t - t_n)$.
Here t_n are time moments when the ice rod crushes at $x = q(t)$; d_n are the lengths of ice blocks that split off, and $H(z)$ stands for the Heaviside step function. The time moments t_n are defined by the condition

$$p(t_n) = p_c, \tag{6}$$

that is, the pressure p in the ice periodically attains a critical level p_c. We introduce the following boundary conditions

$$u(q(t),t) = q(t), \quad u(x,t) \to 0, \; for \; x \to +\infty. \tag{7}$$

The first one is a contact relation between the ice rod and the oscillator, and the second one is a radiation condition at infinity. The initial conditions are given by

$$u(x,0) = \phi_0(x), \quad u_t(x,0) = \phi_1(x), \quad x \in (q(0), \infty), \tag{8}$$

where $\phi_j(x)$ are fast decreasing functions in x for $x \to \infty$.

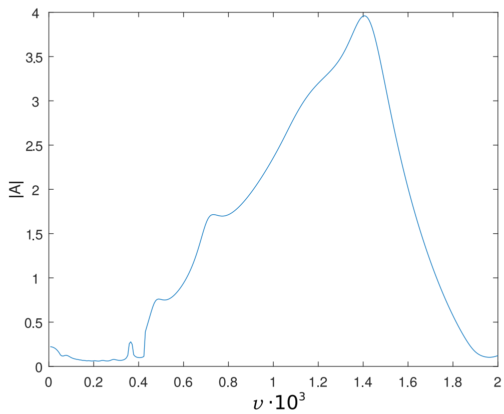

Figure 1: The dependence of the equilibrium amplitude A on the ice rod speed v for the dimensionless parameters $k_0 = 25$, $d = 1$, $\beta = 0.2$, $\bar{\alpha} = 4$, and $\delta_0 = 0.1$.

Conclusions

In this paper, a new model is proposed to describe the interaction between an ice-rod and an oscillator. This model takes into account deformations of the ice floe. The model is analytically investigated by asymptotic methods.It is shown that the contact interaction between the oscillator and the ice rod leads to resonances. The main resonance between the oscillator and the external load occurs when the natural frequency Ω of the oscillator is close to the external load frequency ω. We also find a new mechanism for the oscillator's behavior which can be described by a resonance between the external load, the oscillator, and the part of the ice rod (boundary layer) inducing an oscillator-ice rod interaction during the rod's motion with speed v. We show that ice rod deformation patterns arise which are generated by an interaction between the oscillator and the rod. This ice rod deformation is exponentially decreasing along the ice rod length. We obtain a plot describing how the oscillator's amplitude A depends on the ice velocity v. For some parameter choices this plot shows a significant peak for the amplitude A. For small rod-speeds v we have a small oscillation amplitude A, as well as for large v. The height and width of the peak depend on the system parameters.

References

[1] Matlock, H., Dawkins, W.P., and Panak, J.J. (1971) Analytical Model for Ice Structure Interaction. ASCE Journal of the Engineering Mechanics Division, EM 4, *J. Fluid Mech* **1083**:1083-1092.

[2] Sodhi, D.S.: A theoretical model for Ice-Structure Interaction, Proc. of the OMAE-94 conference, ASME, New York, Vol. IV, pp. 29–34.

[3] G.Huang and P.Liu (2008) A dynamic model for Ice-Induced Vibrations of Structures. Journal of Offshore Mechanics and Arctic Engineering , **011501**: 011501-011506.

[4] H. Hendrikse, A. Metrikine (2015) Interpretation and prediction of ice induced vibrations based on contact area variation. International Journal of Solids and Structures 75 **336** 336-348.

[5] V.Z. Vlasov, N.N. Leont'ev: Beams,plates and shells on elastic foundations, Israel Program for Scientific Translation, Jerusalem, Israel, 1966.