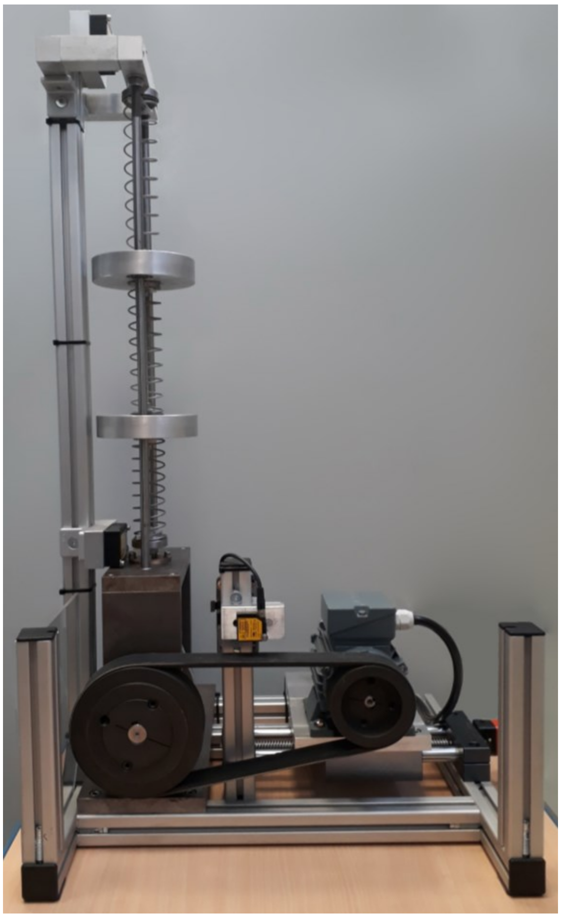
Test Rig with Drive Belt: Modelling and Simulation of Parametrically Excited Vibrations

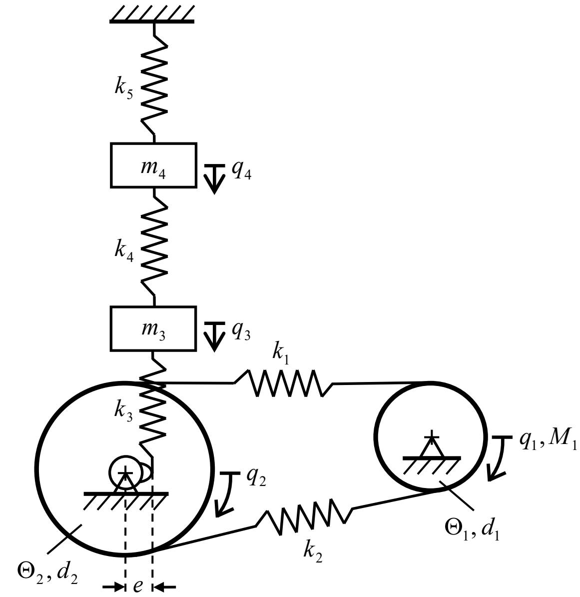
Markus H. E. Messer and Bernhard Gass
Fachbereich Maschinenbau, Mechatronik, Materialtechnologie (Mechanical Engineering), Technische Hochschule Mittelhessen (University of Applied Sciences), Friedberg, Germany

Summary. This paper deals with the modelling and simulation of parametrically excited vibrations in a test rig with a drive belt. The structure is modelled as a dynamical system with time-periodic coefficients and four degrees of freedom (two rotational and two translational degrees of freedom). Stiffness parameters are obtained from experimental load-displacement diagrams, and mass parameters are calculated by hand or by CAD software. The system's proportional damping parameters are derived from measured damping ratios of the structure. Finally, the parametrically excited vibrations are studied numerically by using fourth-order Runge-Kutta method. The simulation results are considered to be very valuable for the upcoming measurements and the analysis of experimental data.

Introduction

Machines with drive belts often exhibit parametrically excited vibrations [1]. The test rig in figure 1 uses a V-ripped L-profile drive belt. For the most part, the drive belt consists of vulcanized rubber material. In addition to that, the drive belt contains a load-carrying tension member (high strength fibres) for transmitting the longitudinal forces. This type of drive belt can be found in many different machines, e.g. sheet-fed offset printing machines. Recent work of one of the authors showed, that the drive belt can be modelled as a spring with time-periodic stiffness. This parametric stiffness excitation leads to numerical results that match very well with measured vibrations [2].





Parameters:
$\Theta_1 = 1.64 \cdot 10^{-3}$ kg m^2, $\Theta_2 = 2.21 \cdot 10^{-2}$ kg m^2,
$m_3 = m_4 = 1.25$ kg, $d_1 = 0.104$ m, $d_2 = 0.164$ m,
$e = 0.005$ m, $k_3 = k_4 = k_5 = 492$ N/m,
$\alpha = 2.45$ s^{-1}, $\beta = 8.37 \cdot 10^{-4}$ s

Drive belt's geometry and stiffness:
$L = 0.991$ m, $h = 0.007$ m, $b = 0.038$ m,

$$k_1 = k_2 = 6.78 \cdot 10^5 \text{ N/m} \cdot \left[1 + 0.175 \cdot \cos\left(\frac{\dot{q}_1 \pi d_1 t}{L}\right)\right]$$

Figure 1 (left): Photograph of the test rig. Figure 2 (right): Sketch and parameters of the dynamical system.

Dynamical system

Figure 2 displays a sketch of the dynamical system. The rotational degrees of freedom of the two belt pulleys are denoted by q_1 and q_2. The two masses m_3 and m_4 have translational degrees of freedom (q_3 and q_4). An asynchronous motor fed from a frequency converter delivers the drive torque M_1 to the small belt pulley. According to [3], the drive belt's stiffness can be different for tight span (k_1) and slack span (k_2). k_3, k_4 and k_5 represent helical compression springs.

Applying Newton's Second Law yields the following equations of motion:

$$\Theta_1\ddot{q}_1 + (k_1 + k_2)\frac{d_1^2}{4}q_1 - (k_1 + k_2)\frac{d_1 d_2}{4}q_2 - M_1 = 0 \tag{1}$$

$$\Theta_2\ddot{q}_2 + (k_1 + k_2)\frac{d_2^2}{4}q_2 - (k_1 + k_2)\frac{d_1 d_2}{4}q_1 + k_3 e^2\sin(q_2)\cos(q_2) - k_3 e\cos(q_2)q_3 = 0 \tag{2}$$

$$m_3\ddot{q}_3 + (k_3 + k_4)q_3 - k_4 q_4 - k_3 e\sin(q_2) = 0 \tag{3}$$

$$m_4\ddot{q}_4 + (k_4 + k_5)q_4 - k_4 q_3 = 0 \tag{4}$$

Kammer experimentally acquired the linear load-displacement diagram for the helical compression springs used in the test rig [4]. This leads to $k_3 = k_4 = k_5 = 492$ N/m. Furthermore, the authors' colleagues Pyttel and Wiesner carried out a tensile test of the drive belt [5]. The measured load-displacement relationship is nonlinear, but linearizing the progressive (hardening) curve at the operating point (belt tension: 278 N) leads to: $k = 6.78 \cdot 10^5$ N/m. In accordance with recent work of one of the authors, the drive belt is modelled as a spring with time-periodic stiffness as follows [2].

$$k_1 = k_2 = k \cdot [1 + \varepsilon \cdot \cos(\omega_{PE} t)] \tag{5}$$

Neglecting slip, the parametric excitation frequency ω_{PE} relates to the drive belt's length L as follows:

$$\omega_{PE} = \frac{\dot{q}_1 \pi d_1}{L} \tag{6}$$

For $\dot{q}_1 = \dot{q}_2 = 0$ (non-rotating belt pulleys) Kammer measured free vibrations of m_3 and m_4 and identified eigenfrequencies (20 rad/s, 34 rad/s) and corresponding damping ratios (0.07, 0.05) [4]. Consistent with [2], proportional damping ($\underline{D} = \alpha\underline{M} + \beta\underline{K}$) is assumed, and α and β are chosen to yield the system's above-mentioned damping ratios.

Simulation of parametrically excited vibrations

Since the drive torque M_1 hasn't been measured yet, the dynamical system is simulated for the scenario $\dot{q}_1 = \text{const.}$ Figure 3 shows simulation results for $\varepsilon = 0.175$, which is in line with [2]. Maximum amplitudes in figure 3 occur at order $0.33 = \pi d_1/L$ (first drive belt order). Simulation results will be compared with impending measurements.

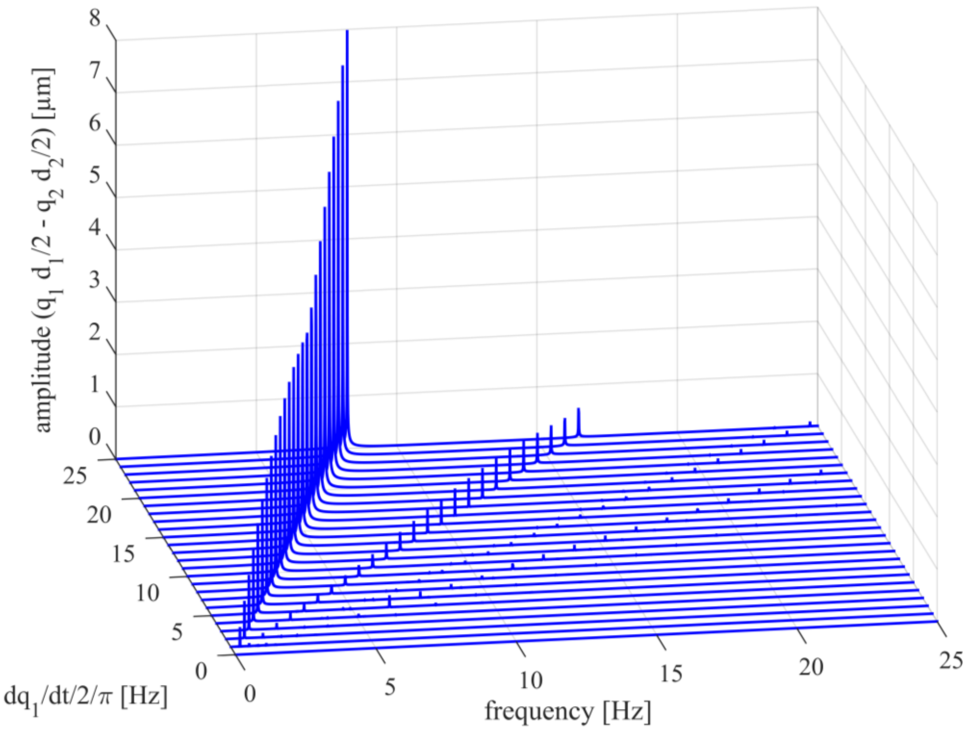


Figure 3: Simulated parametrically excited vibrations

References

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