EXPLOITING NONLINEAR DYNAMICS FOR MANIPULATION OF ACOUSTICALLY LEVITATED PARTICLES

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Summary. This paper describes an experimentally verified local and spatial model of a standing waves acoustic levitation system. By tailoring the waveforms fed to an array of ultrasonic transducers, arranged in a dome formation, individual potential-wells' can be created and altered periodically. The waveforms contain a fast part, geared to produce the nonlinear acoustic field and a slow part, tuned to create slow modulation of the local stiffness and principle parametric excitation. The latter can bring about large controlled oscillations of the levitated objects. Presented is a local, nonlinear model describing the dynamics of an individual acoustically levitated particle in a single potential well. And, for the first time, the dynamics of coupled floating particles tied through the weak nonlinear acoustic field, is shown. Nonlinearity of the acoustic field serves several purposes here: (i) It creates the nonlinear field with potential-wells and hence acoustic levitation (ii) It creates large motions and that produce hopping out of potential wells, given a suitably modulated slow excitation.

Introduction

This paper describes a method to perform fast dynamic manipulation of acoustically levitated particles. The results extend high frequency and high intensity acoustic excitation methods to dynamically manipulate standing and traveling wave patterns affecting particles' motion in 3D space in a controlled manner. Sound pressure levels that are generated in standing waves patterns at around 40 kHz can exceed 140db within a restricted area, giving rise to nonlinear acoustic phenomena. Such nonlinear phenomena can hold and mobilize particles of several millimeter in size at 10-80Hz frequencies. State-of-the-art research enables quasi-static motions of levitated particles by affecting the acoustic field and to slowly move their stable locations. The present work shows experiments where parametric excitation is created by modulating the acoustic field such that particles can be made to oscillated within acoustic potential wells. To achieve these goals, high order nonlinear models are generated, and embedded in a fast (1MHz sampling rate) excitation signals. The main uses of acoustic manipulation are in containerless and micro-gravity processing where contamination is a concern, when mild holding forces are sought and when particles and drops are to be mixed or moved in unison.

Described is the model of slow particle motion and its link to the fast (ultrasonic) acoustic excitation. In addition, some fast videos of a working experimental system are analyzed to produce qualitative and quantitative parameters shading light on the physical phenomenon. If viscosity has a small effect compared with the higher order acoustic radiation forces, the analysis of acoustic levitation devices (ALD), usually considers higher terms beyond the linear approximation of the pressure and particle velocity, but viscosity is often neglected in the initial computations.

Acoustic radiation pressure and force – simplified form

To predict nonlinear acoustic phenomenon creating the levitation, the well-known linear acoustic wave equation (2), does not suffice, and higher order terms should be considered [1,2]. Neglecting dissipation and heat related loss [3,4] for simplicity, one can obtain the fundamental nonlinear effect that predicts the stable acoustic potential-wells in space. These field parameters are used, whereas it is assumed that there is no mean flow in the cavity:

$$p = p_0 + p_1 + p_2 + \dots,\quad \mathbf{u} = \mathbf{u}_0 + \mathbf{u}_1 + \mathbf{u}_2 + \dots = \mathbf{u}_1 + \mathbf{u}_2 + \dots,\quad \rho = \rho_0 + \rho_1 + \rho_2 + \dots . \tag{1}$$

The pressure field comprising the ambient pressure p_0 and high order terms p_1 and p_2, $\mathbf{u}$ is the particle velocity field expended in a similar manner and ρ is the density field comprising the ambient density and higher order terms. In the linear case, the wave equation reduces to

$$\nabla^2 p_1 = \frac{1}{c_0^2}\frac{\partial^2 p_1}{\partial t^2}. \tag{2}$$

In the nonlinear case, the time-averaged acoustic radiation pressure can be calculated from these linear fields [1]:

$$\langle p_2 \rangle = \frac{1}{\rho_0 c_0^2}\langle p_1^2 \rangle - \frac{\rho_0}{2}\langle \mathbf{u}_1 \cdot \mathbf{u}_1 \rangle, \tag{3}$$

whereas c_0 is the sound speed in the medium. The radiation force acting on an object is calculated as:

$$\mathbf{F}_{\text{rad}} = -\int_{S_0} \langle p_2 \rangle \mathbf{n} \mathrm{d}S - \int_{S_0} \rho_0 \langle (\mathbf{n}\cdot\mathbf{u}_1)\mathbf{u}_1 \rangle \mathrm{d}S . \tag{4}$$

whereas S_0 is the object surface while **n** represents the surface normal vector. Viscous effects that create oblique and tangent forces [5] are ignored her. For small (i.e., the radius is much smaller than the acoustic wavelength) rigid spheres, the acoustic radiation force is approximated by:

$$\mathbf{F}_{\text{rad}} = -\nabla U,\quad U = 2\pi R^3\left(\frac{f_1}{3\rho_0 c_0^2}\left\langle \left(p_1^{in}\right)^2 \right\rangle - \frac{f_2 \rho_0}{2}\langle \mathbf{u}_1^{in}\cdot \mathbf{u}_1^{in} \rangle\right),\ f_1 = 1 - \frac{\rho_0 c_0^2}{\rho_p c_p^2},\quad f_2 = 2\left(\frac{\rho_p - \rho_0}{2\rho_p + \rho_0}\right). \tag{5}$$

Here, R, ρ_p and c_p are the sphere radius, density and sound speed in the sphere, $\bullet^{in}$ are computed in the absence of the sphere in the field, and U is called Gor'kov potential [6]. It is now possible to define the total potential energy as $U_{\text{tot}} = U + \underline{U}_{\text{gravity}}$, and levitate objects steadily in the minima of U_{tot}.

Analytical model of a levitated particle subjected to parametric excitation

The dynamics of an acoustically levitated particle is governed by several physical effects: acoustic radiation forces, acoustic streaming, drag force and inertia. To accommodate effects not considered by (5) the following dynamical model for an acoustically levitated particle subjected to PE was developed, were the parameters were estimated experimentally [7].

$$u'' + 2\varepsilon\zeta_1 u' + \varepsilon\zeta_2 u'^2 + \varepsilon^2\zeta_3 u'^3 + \left(1+\varepsilon\gamma\cos(\Omega\tau)\right)\left(u + \varepsilon\kappa_2 u^2 + \varepsilon^2\kappa_3 u^3\right) = \varepsilon\gamma F\cos(\Omega\tau). \tag{6}$$

Equation (6) is dimensionless, where u is the particle position relative to the equilibrium position, ζi are damping coefficients, γ is the parametric excitation (PE) magnitude generated by modulating the voltage provided to the transducer [7], κi are nonlinear stiffness coefficients, Ω is the scaled parametric excitation frequency and F is an external force amplitude. The damping coefficients related terms dissipates energy from the system and can be due to drag, viscosity and acoustic streaming. The paper will include some asymptotic analysis and experimental results.

Contactless particle manipulation via standing wave acoustic levitation - experiments

In standing wave-based levitation, a strong standing wave acoustic field is generated in a confined space, also known as the acoustic cavity. When an object is placed in the cavity it is subjected to an acoustic radiation pressure, the control of this pressure allows manipulation of the object. The ability to control the motion of a floating particles vertically and horizontally is demonstrated by the experimental system depicted in Fig.1.

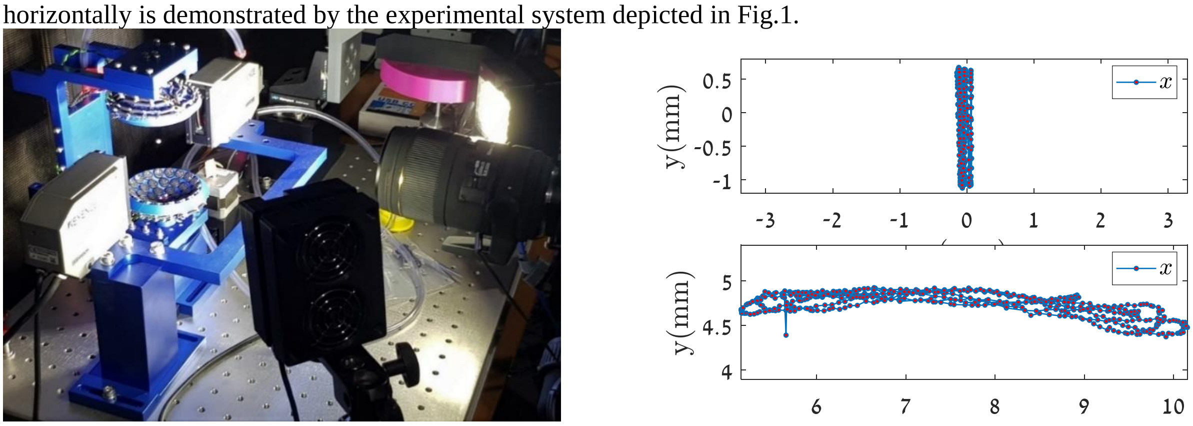

Figure 1. Left: Fast video set-up. Showing single levitated particle and fast Video recording (40,000 fps). Right/top: Fast video particle tracking using (DIC), horizontal oscillations within the potential-well for Ω=16.5 Hz. Right/bottom: vertical oscillation by inducing Ω=31 Hz videos. Links: **horizontal** and **vertical**.)

Another feature not predicted by the Gor'kov's theory is the coupling between levitated particles when the acoustic field is modulated at suitable frequencies as shown below (see also [7]). The frames shown in Figure 2 illustrates that the bottom particle is stationary while the top and 2nd from bottom move in unison. The latter is caused by higher order forces than described by Gor'kov's theory and is one of the proposed topics.

Figure 2. Showing 3 frames from a fast video, spaced about 5 milliseconds apart. Using the experimental system Fig.1. See seconds 7-8 in Link to **VIDEO#1**. For modulating the excitation at 37.83 Hz.

Conclusions

Presented is the analysis and experiments of an acoustic levitation system capable of manipulating levitated particles by means of time varying nonlinear effects. It is shown that previously ignored effects are important for the dynamics of an array of levitated particles. Nonlinear effect are controlled by a model based fast signal processor, showing good agreement with the asymptotic model.

References

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