Multi-mode approximation of VIVs in vertical and horizontal flexible risers

Victoria Kurushina[*,†] and Ekaterina Pavlovskaia[‡]
[*] *Centre for Perspective Research and Innovative Development, Industrial University of Tyumen, Tyumen, 625000, Russia*
[†] *School of Engineering, Newcastle University, Newcastle, NE1 7RU, United Kingdom*
[‡] *Centre for Applied Dynamics Research, University of Aberdeen, King's College, Aberdeen, AB24 3UE, United Kingdom*

Summary. In this study, oscillations of a vertical flexible structure with pinned-pinned ends in uniform flow are modelled and compared with the responses of a horizontal structure. Wake oscillator approach is adopted to simulate vortex-induced forces in the in-line and cross-flow directions and multi-mode approximation of the structural response is developed. Differences in the observed multi-mode lock-in behaviour and hysteretic responses for vertical and horizontal configurations are discussed for the case previously considered in experiments [1].

Introduction

Wake oscillator method allows simplifying calculations of the fluid forces acting on the slender structure vibrating in the fluid flow so that complex phenomenon of vortex-induced vibrations (VIVs) for a variety of structures and case parameters could be investigated with limited computational efforts. In the current study, analysis of vertical pipe vibrations is performed based on the previous investigations of a flexible structure in 1D [2] for the wake oscillators presented in [3].

Flexible riser model

Initially in straight configuration, flexible structure with pinned-pinned ends is considered vibrating in uniform current in 2 dimensions, accounting for the in-line and cross-flow displacements that varies along the length, L. The pipe is modelled as an Euler–Bernoulli beam whereas external fluid forces are described using nonlinear wake oscillator equations. Here, the Krenk-Nielsen oscillator [4] with the frequency doubling coefficients is selected to predict the drag force fluctuations based on the study [3] where the effect of phenomenological oscillators on the performance of the flexible riser model for the horizontal configuration were considered. The obtained coupled system of nonlinear partial differential equations is simplified employing Galerkin–type discretisation to create reduced order model. The resulting ordinary differential equations are solved numerically providing multi-mode approximations of the structure displacements and non-dimensional fluid force coefficients. The obtained model is applicable for both horizontal and vertical configurations and can be written in non-dimensional form for ith mode as:

$$
\begin{aligned}
\ddot{X}_i \quad &+\quad 2a\Omega_R\dot{X}_i+\omega_{ni}^2X_i=\frac{W_w}{Lm_*\omega_0^2}\sum_{n=1}^{N}X_n\Phi_{ni}+\frac{a\Omega_R^2}{i\pi^2St}(1-\cos(i\pi))+\frac{b\Omega_R^2}{4\pi St}w_i-b\Omega_R\sum_{n=1}^{N}\sum_{m=1}^{N}w_n\dot{X}_m\Pi_{nmi}\\
&+\quad \frac{c\Omega_R}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}q_n\dot{Y}_m\Pi_{nmi}+2\pi aSt\sum_{n=1}^{N}\sum_{m=1}^{N}\dot{X}_n\dot{X}_m\Pi_{nmi}+\pi aSt\sum_{n=1}^{N}\sum_{m=1}^{N}\dot{Y}_n\dot{Y}_m\Pi_{nmi},\\
\ddot{Y}_i \quad &+\quad a\Omega_R\dot{Y}_i+\omega_{ni}^2Y_i=\frac{W_w}{Lm_*\omega_0^2}\sum_{n=1}^{N}Y_n\Phi_{ni}-\frac{b}{2}\Omega_R\sum_{n=1}^{N}\sum_{m=1}^{N}w_n\dot{Y}_m\Pi_{nmi}+\\
&+\quad \frac{c}{4\pi St}\Omega_R^2q_i-c\Omega_R\sum_{n=1}^{N}\sum_{m=1}^{N}q_n\dot{X}_m\Pi_{nmi}+2\pi aSt\sum_{n=1}^{N}\sum_{m=1}^{N}\dot{X}_n\dot{Y}_m\Pi_{nmi},\\
\ddot{w}_i \quad &-\quad 2\varepsilon_{x1}\Omega_R\dot{w}_i+2\varepsilon_{x2}\Omega_R\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}\Big(\dot{w}_nw_mw_l\Psi_{nmli}\Big)+2\frac{\varepsilon_{x3}}{\Omega_R}\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}\Big(\dot{w}_n\dot{w}_m\dot{w}_l\Psi_{nmli}\Big)+4\Omega_R^2w_i=A_x\ddot{X}_i,\\
\ddot{q}_i \quad &-\quad \varepsilon_y\Omega_R\dot{q}_i+\varepsilon_y\Omega_R\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}\Big(\dot{q}_nq_mq_l\Psi_{nmli}\Big)+\Omega_R^2q_i=A_y\ddot{Y}_i,
\end{aligned}
$$

where X_i and Y_i are in-line and cross-flow displacements multipliers, w_i and q_i are wake coefficients multipliers, Ω_R is vortex shedding frequency, St is Strouhal number, $\varepsilon_{x1}, \varepsilon_{x2}, \varepsilon_{x3}, \varepsilon_y$ are dimensionless wake oscillator damping coefficients, A_x, A_y are empirical coupling coefficients, Φ_{ni}, Π_{nmi} and Ψ_{nmli} are dimensionless coefficients obtained during the discretisation procedure due to the mode interaction, a, b, c are dimensionless coefficients depending on the initial drag, fluctuating drag and lift coefficients respectively, N is total number of modes considered, ω_{ni} is the ith natural frequency, ω_0 is reference frequency, m_* is mass per unit length (including structural mass and fluid added mass), W_w is weight of structure per unit length. Then the displacements and wake coefficients are calculated as

$$
X(\zeta,\tau)=\sum_{n=1}^{N}X_n(\tau)\widetilde{X}_n(\zeta);\quad Y(\zeta,\tau)=\sum_{n=1}^{N}Y_n(\tau)\widetilde{Y}_n(\zeta);\quad w(\zeta,\tau)=\sum_{n=1}^{N}w_n(\tau)\widetilde{w}_n(\zeta);\quad q(\zeta,\tau)=\sum_{n=1}^{N}q_n(\tau)\widetilde{q}_n(\zeta).
$$

where τ is non-dimensional time and the sinusoidal functions $\widetilde{X}_n(\zeta), \widetilde{Y}_n(\zeta), \widetilde{w}_n(\zeta), \widetilde{q}_n(\zeta)$ depending on the location along the beam $\zeta = \frac{z}{L}$ are used. Here, z is the longitudinal coordinate, and $z = 0$ indicates the bottom for the vertical configuration.

For our analysis, the configuration of the riser was selected to match the case experimentally investigated in [1] where a flexible tube with internal fluid with the weight of 2.97 N/m, length of 2.92 m, diameter of 0.018 m, and applied tension of 147 N was considered in the Reynolds number range from around 1780 to 14800. The other relevant parameters were the mass ratio of 1.17 and damping ratio of 0.025.

Results

Comparison of the lock-in curves calculated for in-line and cross-flow displacement amplitudes for the independent velocity points and under decreasing and increasing flow velocity is performed in this study for the horizontal and vertical risers, as illustrated in Figs 1a and 1b. Hysteretic behaviour is observed in the valleys between the lock-in peaks. The co-existing solutions obtained with decreasing and increasing velocities indicate significant differences in the displacement magnitudes. Figures 1c and 1d demonstrate displacement amplitudes of three co-existing solutions along the riser length for a chosen value of the reduced velocity of $U_R = 10.48$. Analysis of the obtained responses reveal a different number of modes observed in the in-line and cross-flow directions. An apparent asymmetry towards higher amplitudes in the response of the vertical structure is noticeable below $\zeta = 0.50$.

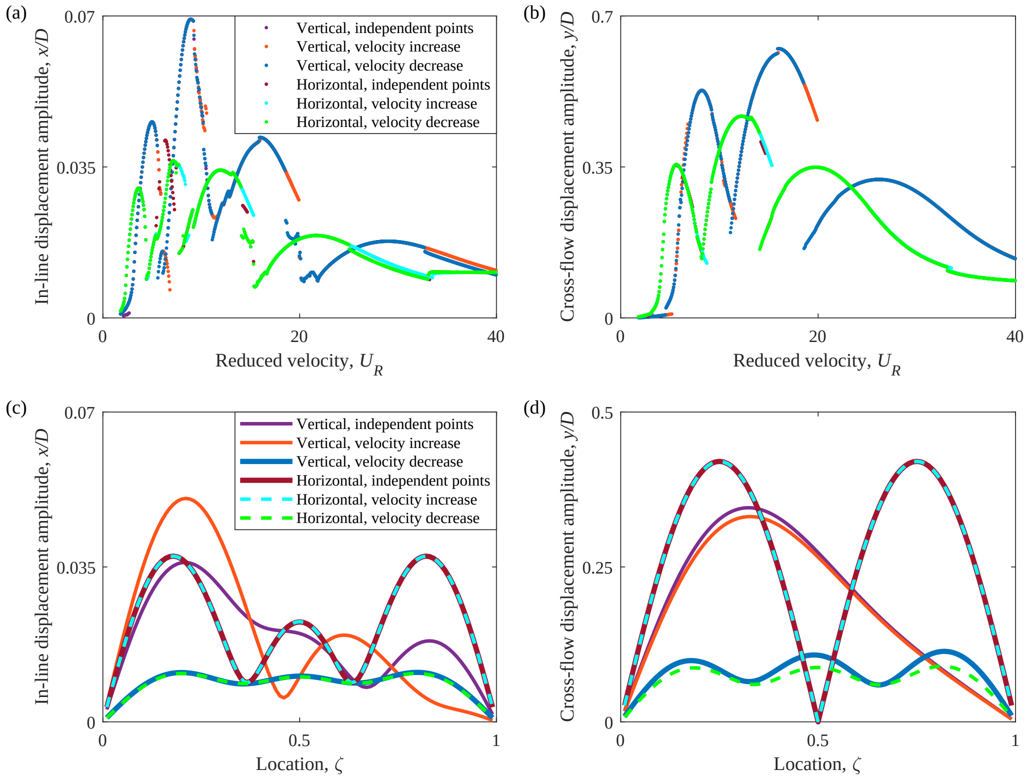

Figure 1: Dynamics of a vertical and horizontal flexible riser for the range of reduced velocity predicted by the 3 mode approximation: (a) in-line and (b) cross-flow displacement multi-mode lock-in at $\zeta = 0.25$; (c) in-line and (d) cross-flow displacement amplitude variation along the riser length at the reduced velocity of $U_R = 10.48$.

Conclusions

This study investigates the differences in the predicted behaviour of a flexible structure in uniform flow for vertical and horizontal configurations under increasing and decreasing flow velocity, and also in the limited case, when the structure starts oscillating from zero initial conditions for each flow velocity. Vertical model shows the higher amplitudes of displacement from the initial position than the horizontal structure in general. Vertical structure also demonstrates a large asymmetry of the response, a more notable contribution of the nearby modes at the nodal points and a slight delay in the peak occurrence in the reduced velocity range, compared to the horizontally positioned one.

References

[1] Sanaati B., Kato N. (2012) A study on the effects of axial stiffness and pre-tension on VIV dynamics of a flexible cylinder in uniform cross-flow. *Applied Ocean Research* **37**:198-210.

[2] Pavlovskaia E., Keber M., Postnikov A., Reddington K., Wiercigroch M. (2016) Multi-modes approach to modelling of vortex-induced vibration. *International Journal of Non-Linear Mechanics* **80**: 40-51.

[3] Kurushina V., Pavlovskaia E., Wiercigroch M. (2020) Modelling of flexible structure VIV in a uniform flow. *International Journal of Engineering Science* **150**: 103211.

[4] Krenk S., Nielsen, S.R.K. (1999) Energy balanced double oscillator model for vortex-induced vibrations. *Journal of Engineering Mechanics* **125(3)**:263-271.