Vibrations of a vertical flexible riser in sheared flow

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Summary. Vortex-induced vibrations of a flexible pipe, with pinned-pinned ends and straight initial configuration, are considered in this work in a number of linear and nonlinear velocity profiles, varying along the length of structure. Krenk-Nielsen [1] and Van der Pol wake oscillators are employed in order to simulate fluctuations of the fluid forces, while the structure is modelled using the Euler-Bernoulli beam. Trajectories, time histories, multi-mode lock-in are studied using the case parameters, previously investigated by [2].

Introduction

Vibrations of flexible structures remain a vital problem for the safety of oil and gas subsea production systems. Application of fluid oscillators is one of the modern ways to improve the current prediction capabilities and study the complex multi-mode vibration mechanism. The focus of this work is on the near-resonant velocity profiles and resulting behaviour of the vertical flexible structure.

Flexible riser model

The structure with pinned-pinned ends, previously studied in uniform flow in 2D in [3], is subjected to sheared flow, and the approximate solution is obtained using the method described in [4]. Approximation for sheared flow is developed using integration of the velocity profile along the length of the riser. The reduced order non-dimensional model for *i*th mode is as follows:

$$
\begin{aligned}
&\ddot{X}_i + 2a\sum_{n=1}^{N}\dot{X}_n\Theta_{ni} + \omega_{rn}^2 X_i = \frac{W_w}{Lm_*\omega_0^2}\sum_{n=1}^{N}X_n\Phi_{ni} + \frac{a}{2\pi St}\Lambda_i + \frac{b}{4\pi St}\sum_{n=1}^{N}w_n\Gamma_{ni} - b\sum_{n=1}^{N}\sum_{m=1}^{N}w_n\dot{X}_m\Delta_{nmi} + \qquad (1)\\
&+\frac{c}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}q_n\dot{Y}_m\Delta_{nmi} + 2\pi Sta\sum_{n=1}^{N}\sum_{m=1}^{N}\dot{X}_n\dot{X}_m\Pi_{nmi} + \pi Sta\sum_{n=1}^{N}\sum_{m=1}^{N}\dot{Y}_n\dot{Y}_m\Pi_{nmi};\\
&\ddot{Y}_i + a\sum_{n=1}^{N}\dot{Y}_n\Theta_{ni} + \omega_{rn}^2 Y_i = \frac{W_w}{Lm_*\omega_0^2}\sum_{n=1}^{N}Y_n\Phi_{ni} - \frac{b}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}w_n\dot{Y}_m\Delta_{nmi} + \frac{c}{4\pi St}\sum_{n=1}^{N}q_n\Gamma_{ni} -\\
&-c\sum_{n=1}^{N}\sum_{m=1}^{N}q_n\dot{X}_m\Delta_{nmi} + 2\pi Sta\sum_{n=1}^{N}\sum_{n=1}^{m}\dot{X}_n\dot{Y}_m\Pi_{nmi};\\
&\ddot{w}_i - 2\varepsilon_{x1}\sum_{n=1}^{N}\dot{w}_n\Theta_{ni} + 2\varepsilon_{x2}\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}\dot{w}_n w_m w_l \Xi_{nmli} + 2\varepsilon_{x3}\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}\dot{w}_n\dot{w}_m\dot{w}_l B_{nmli} + 4\sum_{n=1}^{N}w_n\Gamma_{ni} = A_x\ddot{X}_i;\\
&\ddot{q}_i - \varepsilon_y\sum_{n=1}^{N}\dot{q}_n\Theta_{ni} + \varepsilon_y\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}\dot{q}_n q_m q_l \Xi_{nmli} + \sum_{n=1}^{N}q_n\Gamma_{ni} = A_y\ddot{Y}_i,
\end{aligned}
$$

where

$$
X(\zeta,\tau)=\sum_{n=1}^{N}X_n(\tau)\widetilde{X}_n(\zeta);\quad Y(\zeta,\tau)=\sum_{n=1}^{N}Y_n(\tau)\widetilde{Y}_n(\zeta);\quad w(\zeta,\tau)=\sum_{n=1}^{N}w_n(\tau)\widetilde{w}_n(\zeta);\quad q(\zeta,\tau)=\sum_{n=1}^{N}q_n(\tau)\widetilde{q}_n(\zeta).
$$

Here, τ is non-dimensional time, $\zeta = \frac{z}{L}$ is location of considered cross-section of the beam (where z represents the axial coordinate), $\widetilde{X}_n(\zeta), \widetilde{Y}_n(\zeta), \widetilde{w}_n(\zeta), \widetilde{q}_n(\zeta)$ are sinusoidal functions, X_i and Y_i constitute in-line and cross-flow displacements multipliers, w_i and q_i are wake coefficients multipliers, Ω_R is vortex shedding frequency, St is Strouhal number, $\varepsilon_{x1}, \varepsilon_{x2}, \varepsilon_{x3}, \varepsilon_y$ are dimensionless wake oscillator damping coefficients, A_x, A_y are empirical coupling coefficients, a, b, c are dimensionless coefficients depending on the initial drag, fluctuating drag and lift coefficients respectively, N is total number of modes considered, ω_{ni} is the *i*th natural frequency, ω_0 is reference frequency, m_* is mass per unit length (including structural mass and fluid added mass), W_w is weight of structure per unit length. Dimensionless coefficients Λ_i, Θ_{ni}, Φ_{ni}, Γ_{ni}, Δ_{nmi}, Π_{nmi}, B_{nmli}, Ξ_{nmli} represent interaction of modes with numbers n, m, l. Among them, Λ_i, Θ_{ni}, Γ_{ni}, Δ_{nmi}, B_{nmli}, Ξ_{nmli} constitute the difference with the uniform flow model [3] and account for the velocity/vortex shedding frequency profile integrated along ζ, e.g. $\Theta_{ni} = 2\int_0^1 \left[sin(n\pi\zeta)sin(i\pi\zeta)StU_R(\zeta)\right]d\zeta$. In this work, the dynamics of the vertical flexible structure is studied using 5 mode approximation, where reduced velocity is defined based on the first natural frequency measured in [2].

Results

Five sheared flow velocity profiles were considered in the current study where the flow velocities lie in the proximity of the first few natural frequencies of the flexible structure, including reduced velocities $U_R = 5.0$ and $U_R = 10.0$, as shown in Fig. 1a. In this example, constant velocity profile with $U_R = 5.0$ results in the highest in-line and cross-flow displacement amplitudes observed in the bottom part of the structure, as illustrated in Figs 1c and 1d. Structural motion under this constant uniform velocity and under the parabolic profile with the maximum reduced velocity $U_R = 5.0$ show response of 4 modes. The parabolic profile with the maximum reduced velocity $U_R = 10.0$ leads to the most symmetric displacement amplitudes, relatively $\zeta = 0.50$ with three more significant medium peaks. Linearly varying profiles result in the increased displacements in the cross-sections subjected to the increased flow velocity, however, the profile with the velocity growing towards the top of structure leads to higher maximum amplitudes in both in-line and cross-flow directions. Fig. 1b provides two samples of multi-frequency cross-flow displacement signals at the cross-section of $\zeta = 0.33$ with the largest amplitudes demonstrated under the flow of constant velocity.

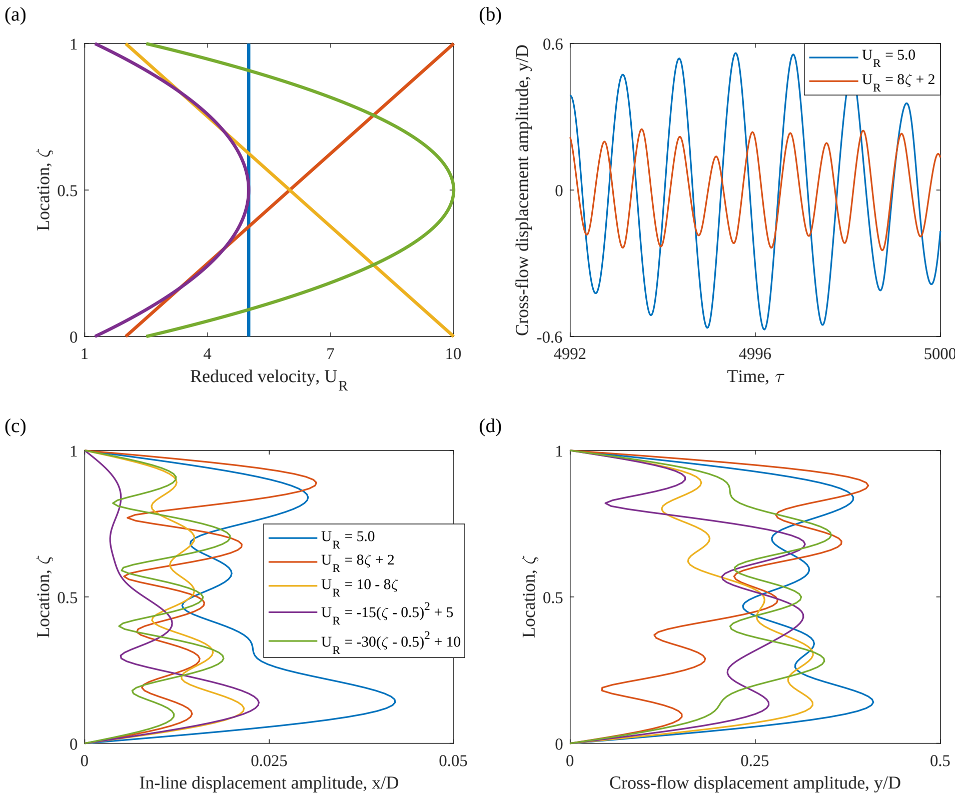

Figure 1: Dynamics of a vertical flexible structure predicted by the 5 mode approximation: (a) five considered velocity profiles shown in terms of the reduced velocity and the length of structure; (b) time histories of the largest and smallest cross-flow displacement amplitudes at $\zeta = 0.33$; (c) in-line and (d) cross-flow displacement amplitude variation along the length of structure.

Conclusions

Results of the flexible structure dynamics simulations for a number of linear and nonlinear sheared flow profiles are presented in this work, using 5 mode approximation obtained in a similar way as was done in the previous studies for uniform flows [3, 4]. The novelty of this model formulation is in the coefficients representing mode interaction and the integrated velocity profile. Multi-mode lock-in is considered in details, and the highest displacement amplitudes are predicted to occur under the influence of linear velocity profiles including the limited cases of uniform flows.

References

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