Stability of Nonlinear Normal Modes under Stochastic Excitation

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Summary. Two-DOF nonlinear system under white noise excitation is considered. It is assumed that the system allows from two up to four nonlinear normal modes (NNMs) with rectilinear trajectories in the system configuration space. Influence of the random excitation to the NNMs stability is analyzed by using the analytical-numerical test, which is an implication of the well-known Lyapunov stability criterion. Boundary of the stability/ instability regions is obtained in place of two parameters, namely, a connection between partial oscillators, and the excitation intensity. Stability of the NNMs under deterministic chaos excitation is also considered.

Introduction

Investigation of nonlinear normal modes (NNMs) is an important part of general analysis of dynamical systems. Different theoretical aspects of the NNMs theory and applications of the theory are presented in numerous publications, in particular, in review [1]. NNMs having rectilinear trajectories in configuration space (so-called similar nonlinear normal modes) were first found in some essential nonlinear systems by Rosenberg [2].

Numerous publications are dedicated to investigation of behavior of dynamical systems under stochastic excitation. In this regard, different theoretical and numerical procedures are developed. We cite here only few publications on the subject, namely, books [3-5]. In [6] the dynamics of aerospace vehicles and/or other structures subject to random excitation is investigated using a reduced order model in terms of its nonlinear normal modes. Evolution of NNMs is studied by the continuation algorithm and presented in the plot "energy-frequency".

Here some numerical-analytical test [7] which is a consequence of the well-know Lyapunov criterion of stability is used to analyze a stability of NNMs in two-DOF nonlinear system under white noise excitation. We assume that the system allows from nonlinear normal modes with rectilinear trajectories in the system configuration space. We analyze also a stability of the NNMs under deterministic chaos excitation.

Principal model under consideration

The following two-DOF nonlinear system under stochastic excitation is considered:

$$\begin{cases} \ddot{x}_1 + x_1^3 + \gamma\left(x_1 - x_2\right)^3 = \varepsilon\xi_1\left(t\right), \\ \ddot{x}_2 + x_2^3 + \gamma\left(x_2 - x_1\right)^3 = \varepsilon\xi_2\left(t\right), \end{cases} \tag{1}$$

where ε is the small parameter, and the functions $\xi_1(t)$, $\xi_2(t)$ represent stochastic excitation. When $\varepsilon=0$ the system allows so-called similar NNMs in the form $x_2 = cx_1$, where c is the modal constant. We can note that the equations (1) always allow two similar NNMs, where $c_{1,2} = \pm 1$. The solution $c_1 = +1$ corresponds to the in-phase nonlinear normal mode, and the solution $c_2 = -1$ corresponds to the out-of-phase (anti-phase) one. We consider the external excitations which save the in-phase and out-of-phase NNMs but can change their stability and point of bifurcation.

Test of stability as implication of the Lyapunov stability criterion

Consider the well-known Lyapunov definition of stability which can be presented in the simplest variant as the following: the solution $y = 0$ is stable if for any positive ε there a positive δ exists such that for all $y_0 \in N_\delta(0)$ and $t \geq 0$ we have the following: $y(t) \in N_\varepsilon(t)$ where stated neighborhoods can be determined using norms in some functional space. Here the variable y has meaning of variation with respect to the solution studied for stability. Later a modulus of the function is used as the norm. One introduces a relation between the quantity ε and the initial value of the variable y; it corresponds to a sense of the definition by Lyapunov when the initial variations must not tend to zero. Let

$$\varepsilon = \rho\left|y_0\right| \leq \rho\delta \ \ (\rho = const) \tag{2}$$

We can note that a value of δ is not arbitrarily small; it corresponds to essence of the Lyapunov definition because in this definition the initial values of variations cannot tend to zero. One has from the Lyapunov stability definition that $\left|y(t)\right| \leq \rho\left|y(0)\right|$ for a case of stability.

We introduce the following test for the system under consideration [6]: Instability of the solution $y = 0$ is fixed if

$$\left|y(t)\right| \geq \rho \ \left|y(0)\right| \qquad \left(0 \leq t \leq T\right). \tag{3}$$

In contrast to the Lyapunov definition a time of numerical calculations T is limited in the test (3). It is necessary to discuss a choice of values of ρ and T. There is some arbitrariness in a choice of the value ρ. In fact, in the instability region the variations leave the solution ε-neighborhood under increase of t for any choice of the parameter ρ. We can choose, for example, ρ = 10. One discusses now a choice of the constant T. Note that all concrete calculations are realized by using the standard Runge-Kutta procedure. These calculations are made at points on some chosen mesh of the system parameter space. Calculations are conducted as long as boundaries of stability/instability regions in a chosen scale on the system parameter space are variable. This is a principal criterion for the choice of the calculation time T [7]. Taking into account a specific character of the stability problem under stochastic excitation, we are introduced some important modification to the proposed test (3). Namely, we can observe that in the last case some variations can leave the ε-neighborhood and then return back to one. Thus, we will allow that some small part of variations (not more than 10 percent) is out of the neighborhood during each fixed interval of time.

Stability of the similar NNMs: results of simulation

To investigate the stability/instability of the in-phase and out-of-phase nonlinear normal modes, we pass to the systems in variations by means of change of variables in (1), namely:

for the out-of-phase NNM $x_1 = (u-v)/\sqrt{2},\ x_2 = (u+v)/\sqrt{2}$, $u = 0 + \Delta u, v = v_0 + \Delta v$, $\xi_2 = -\xi_1$;

for the in-phase NNM $x_1 = (u-v)/\sqrt{2},\ x_2 = (u+v)/\sqrt{2}$, $u = u_0 + \Delta u, v = 0 + \Delta v$, $\xi_2 = \xi_1$.

Some results of numerical simulation for the presented NNM stability problem by using the stability criterion (3) are presented. Figs. 1 and 2 demonstrate boundaries of stability/instability of the out-of-phase nonlinear normal mode for different values of parameters. Figs. 3 and 4 show a behavior in time of the out-of-phase NNM simulated for different values of the connection parameter: γ=0.24 and γ=0.26, correspondingly. Fig.5 shows boundaries of the stability/ instability regions in the place (γ,ε). Note that such results are obtained for four different kinds of stochastic excitation.

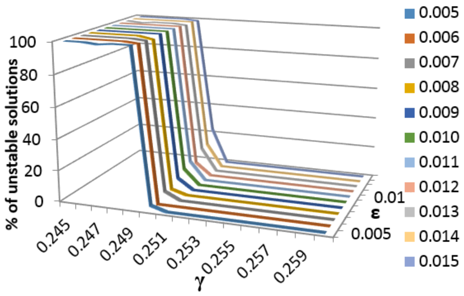

Figure 1

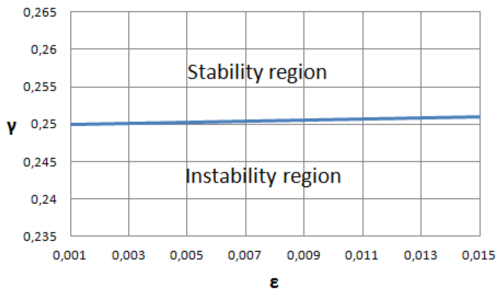

Figure 2

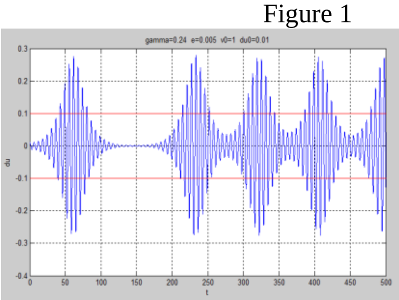

Figure 3

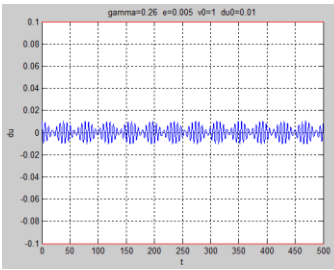

Figure 4

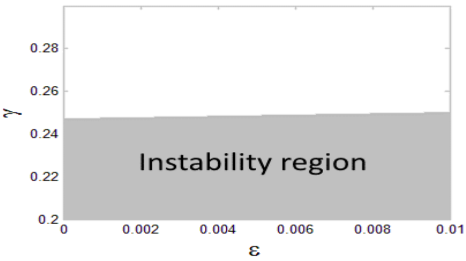

Figure 5

Stability of NNMs under excitation in the form of determined chaos

Determined chaotic excitation can be obtained, in particular, by solution of the non-autonomous Duffing equation for some values of the equation parameters. Numerical simulation shows that excitation in the form of the determined chaos does not influence to stability of the NNMs in the system under consideration.

Conclusions

It is shown that the analytical-numerical test, which is an implication of the Lyapunov stability criterion, can be successfully used in analysis of stability of NNMs in two-DOF system under white noise excitation. Boundary of the stability/ instability regions is obtained in place of the system parameters, including parameter of the excitation intensity. It is shown also that the deterministic chaos excitation does not affect to the NNMs stability.

References

[1] Mikhlin Yu.V., Avramov K.V. (2010) Nonlinear normal modes for vibrating mechanical systems. Review of theoretical developments, *Appl. Mech. Rev*. **63**: 4–20.
[2] Rosenberg, R. (1966) Nonlinear vibrations of systems with many degrees of freedom. *Adv. Appl. Mech*. **9:** 156–243.
[3] Schuss Z. (2010) Theory and Applications of Stochastic Processes, Vol. 170 of Applied Mathematical Sciences. Springer, NY.
[4] Roberts J. B., Spanos P. D. (2003) Random vibration and statistical linearization, Dover Publications.
[5] Li J., Chen J. (2009) Stochastic dynamics of structures. Wiley, NY.
[6] Schoneman J.D., Allen M.S. (2017) Relationships between nonlinear normal modes and response to random inputs. *Mechanical Systems and Signal Processing* **84:** 184-199.
[7] Mikhlin Yu.V., Shmatko T.V., Manucharyan G.V. (2004) Lyapunov definition and stability of regular or chaotic vibration modes in systems with several equilibrium positions. *Computer & Structures* **82:** 2733–2742.