Breather arrest in the damped chains with substantially nonlinear coupling

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Summary. We explore the breather propagation in a damped oscillatory chain with substantially nonlinear (non-linearizable) nearest-neighbour coupling. It is demonstrated that the combination of the damping and substantially nonlinear coupling leads to rather unusual two-stage pattern of the breather propagation. The first stage occurs at a finite fragment of the chain and is characterized by power-law decay of the breather amplitude. The second stage is featured by extremely small breather amplitudes that decay hyper-exponentially. Thus, one can speak about finite penetration depth of the breather – breather arrest. Specific models considered are the chains of damped linear oscillators with Hertzian or purely cubic nonlinear contact between the nearest neighbours. The effect of the initial excitation and of the viscous damping on the breather penetration depth is explored and approximate scaling relationships between these two parameters are established. These results are then rationalized by considering a simplified model of two damped linear oscillators coupled by strongly nonlinear springs. By using an approximate analytic procedure, we demonstrate that the initial excitation of one of these oscillators results in a finite number of beating cycles in the system. Then, the beating cycle in this system of two coupled oscillators is associated with the passage of the discrete breather between the neighbouring sites in the chain. Somewhat surprisingly, this simplified model reliably predicts the main features of the breather arrest in the chain of oscillators. Generalization for arbitrary coupling function and effect of pre-compression are also discussed.

Phenomenon of the breather arrest

We consider a chain of linear forced-damped osillators coupled by essentially nonlinear springs the model is demonstrated in Figure 1.

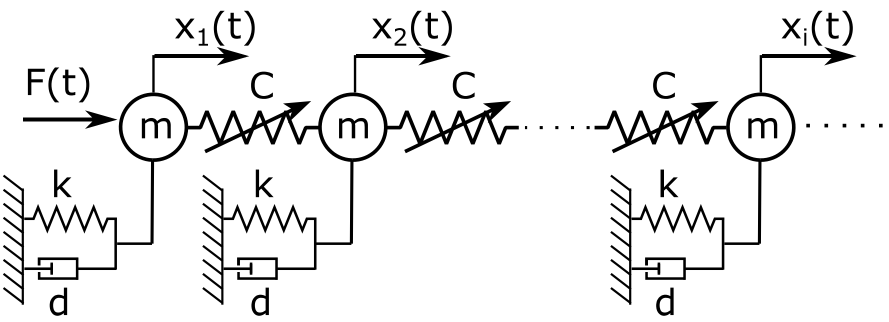

Figure 1. Sketch of the system

The presence of local damping with impulsive forcing inevitably leads to the energy dissipation and therefore decay of the amplitude of the breather propagating along the chain. In the case of linear viscous damping, one intuitively expects that this amplitude decay will be exponential, thus defining characteristic space scale of the decay. However, if the coupling between the neighbours is essentially nonlinear (i.e. non-linearizable), then, due to peculiar interaction between these two factors (essential nonlinearity and damping) one observes an interesting phenomenon of *breather arrest* (BA). The latter is defined as abrupt switch from power-law to hyper-exponential decay of the maximum breather amplitude, leading to a negligibly small amplitude after penetration to *finite* depth in the lattice. Typical dependence of the breather amplitude on the site number (for the case of Hertzian contacts) is presented in Figure 2.

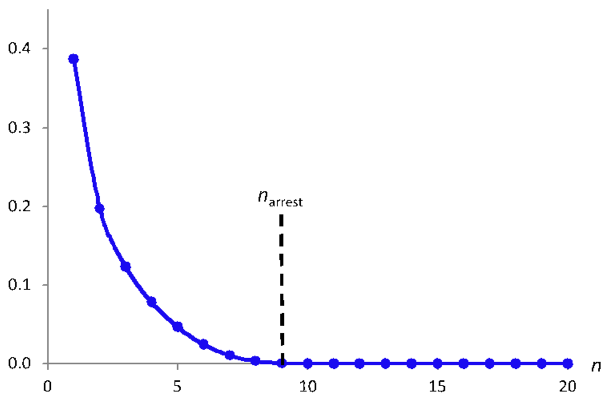

Figure 2. Typical dependence of the breater amplitude on the site.

Numerical simulations demonstrate that the breather penetration depth n_{arrest} scales with initial velocity A and damping coefficient λ according to the approximate relationship:

$$n_{\text{arrest}} \approx A^k \lambda^l \tag{1}$$

In the case of Hertzian contact, one obtains $k = 0.466$ and $l = -0.931$.

Reduced-order model

To assess the BA phenomenon from theoretical viewpoint, and to rationalize the numeric findings, we consider a simplified model that mimics the breather propagation in the chain with strongly nonlinear coupling. This simplification seems to be viable due to extreme localization of the breather in this specific chain. Therefore, it is possible to adopt, in the crudest approximation, that the breather propagation can be understood as a sequence of energy transfers between subsequent particles. Moreover, due to the strong localization, it is possible to assume that each such transfer involves only two neighbouring particles. Thus, the simplified model will consist of two identical oscillators, grounded through pairs of linear springs and viscous dampers, and with essentially nonlinear coupling, as presented in Figure 3.

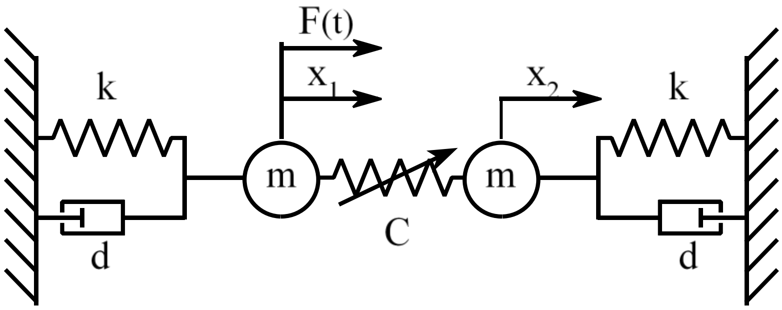

Figure 3. Sketch of the simplified model.

When one of the particles is excited with sufficient amplitude, the beating motion occurs. Each beat is associated with the propagation of the breather by one particle. It is possible to demonstrate that the number of such beats will be finite, due to the strong nonlinearity of the coupling. Then, one can obtain the following estimations for the breather penetration depth, and for the dependence of the breather amplitude on the site number before the arrest:

$$n_{\text{arrest}} \approx \frac{A^{\varepsilon-1}}{\lambda}, \quad A_{\max}(n) \approx A\left(1-\frac{n}{n_{\text{arrest}}}\right)^{\frac{1}{\varepsilon-1}} \tag{2}$$

Here ε is the power of the substantially nonlinear power-law force. These estimations are in good correspondence with numeric findings. If initial pre-compression exists, the asymptotic evaluations (2) are no more valid, but the two-stage pattern of the breather propagation preserves itself, as demonstrated in Figure 4.

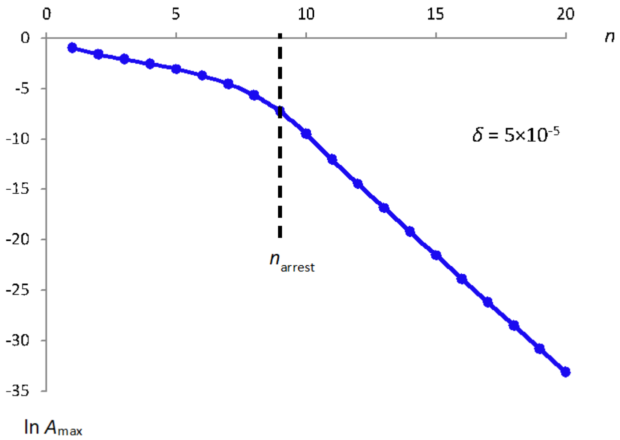

Figure 4 Two-stage breather propagation in granular chain with pre-compression.

These results point on generic character of the observed propagation patterns in damped systems with substantially nonlinear coupling

References

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[2] Strozzi M., Gendelman, O.V. (2019) Breather arrest in a chain of damped oscillators with Hertzian contact**,** arXiv:1907.12462