Attractor Targeting by Dual-frequency Driving

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Summary. A control technique to continuously drive a non-linear, harmonically excited oscillator between different kinds of periodic orbits is presented. The basis of the method is a temporary dual-frequency driving of the system. Results show that two periodic orbits existing at two different frequency values (single frequency driving) having arbitrary periodicity can be smoothly transformed into each other. The method is a proper tuning of the excitation amplitudes of a dual-frequency driving combining the two corresponding frequencies. The requirements are the suitable choice of the frequency pair and the matching of the torsion numbers of the bounding bifurcation points of the periodic orbits.

Introduction

In non-linear systems, multiple domains of periodic attractors might exist in a given parameter space [1]. These domains can overlap each other meaning that at their union, the system is even multi-stable [2]. The different stable solutions usually represent different system performances; for instance, a chemical reactor can have different chemical yield. Thus, it is important to be able to drive a system onto a desired stable state in its parameter space. The main aim of this study is to propose a technique that is suitable to smoothly drive a system from one periodic domain to another.
The method works for harmonically driven non-linear oscillators in the parameter plane of its driving amplitude A_1 and frequency ω_1. The technique is based on the addition of a second harmonic component to the driving with amplitude A_2 and frequency ω_2. With a proper tuning of the driving amplitudes, a periodic attractor exists at ω_1 with $A_2 = 0$ can be smoothly transformed onto another periodic orbit exists at ω_2 with $A_1 = 0$. That is, the beginning and the end of the transformation is a single frequency driven system, and the intermediate states have dual-frequency driving. Throughout the rest of the paper, the requirements of the transformation possibilities are discussed in general; and an example is presented based on the Keller–Miksis equation that is a second order ordinary differential equation describing the radial pulsation of a single spherical bubble [3].

The control technique

For simplicity, let us consider dual-frequency driven second order non-linear oscillators written as

$$\dot{x}_1 = f(x_1, x_2), \tag{1}$$
$$\dot{x}_2 = g(x_1, x_2) + A_1 \cos\omega_1 t + A_2 \cos\omega_2 t. \tag{2}$$

Assume that at fixed frequency ω_1, there is a domain (section) of periodic solution with period p_1 in the A_1 parameter line (the amplitude of the second component is $A_2 = 0$). Similarly, assume that there is another segment of periodic solution with period p_2 in the A_2 parameter line ($A_1 = 0$) at frequency value ω_2 different from ω_1. Both periodic segments are bounded by bifurcation points having torsion numbers q [4]. The schematic draw in Fig. 1A demonstrates such an example in the parameter plane of the excitation amplitudes A_1 and A_2. There are two requirements for the existence of a set of solutions that connects the two segments of these periodic orbits (blue and red lines in Fig. 1A). Without a detailed derivation, the first condition is that the ratio of the periods and ratio of the employed frequencies must be equal:

$$\frac{p_1}{p_2} = \frac{\omega_1}{\omega_2}. \tag{3}$$

It must be stressed that periods of the obits p_1 and p_2 are defined according to the period of the single frequency excitation $T_1 = 2\pi/\omega_1$ and $T_2 = 2\pi/\omega_2$, respectively. If the condition given by Eq. (3) holds, the periods of the two kinds of orbits presented in Fig. 1A by the blue and red lines become equal via employing the period of the dual-frequency driving T as a global Poincaré section. In this case, codimension-2 bifurcation curves might exists that connect the bifurcation point pairs (q_{11}, q_{21}) and (q_{12}, q_{22}) (see the black curves in Fig. 1A). For such an existence, the connected torsion numbers must be equal representing topologically the same local flow of the vector field along the black codimension-2 curves:

$$q_{11} = q_{21}, \tag{4}$$
$$q_{12} = q_{22}. \tag{5}$$

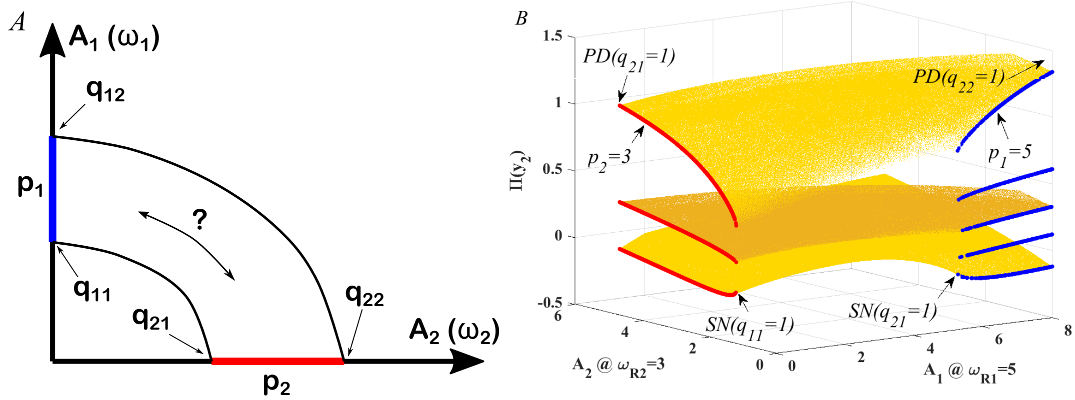

Figure 1: Panel A: Schematic draw of the transformation possibility between two periodic orbits with arbitrary periodicity in the parameter plane of the amplitudes of the dual-frequency driving. Panel B: Transformation between period-3 and period-5 orbits through the yellow surfaces. The employed model is the dual-frequency driven Keller–Miksis equation being a second order ordinary differential equation.

The control technique is demonstrated in Fig. 1B employing the Keller–Miksis equation being a second order non-linear oscillator. For the details of the equation, the reader is referred to the review paper [3]. The frequency combination used is $\omega_{R1} = \omega_1/\omega_0 = 5$ and $\omega_{R2} = \omega_2/\omega_0 = 3$, where ω_0 is the linear resonance frequency of the system. The periods of the orbits studied are $p_1 = 5$ and $p_2 = 3$. Thus, the condition given by Eq. (3) holds. In this figure, the saddle-node and the period doubling bifurcation points are marked by SN and PD respectively. Their torsion numbers are all equal: $q_{11} = q_{12} = q_{21} = q_{22} = 1$ fulfilling also the second requirement defined via Eqs. (4)-(5). The periodic orbits corresponding to the single frequency driving are highlighted by the blue (period-5 at $\omega_{R1} = 5$) and red (period-3 at $\omega_{R2} = 3$) curves. The yellow surfaces represent a set of periodic orbits connecting the period-3 and the period-5 orbits. Therefore, with a proper tuning of the amplitudes A_1 and A_2 of the dual-frequency driving, the system can be smoothly transformed between the period-3 orbits lying on the red curves and the period-5 attractors represented by the blue curves. Observe that the dual-frequency driving is temporary and that the initial and final state of the transformation relate to single frequency driving with different frequencies. It must be emphasized that with a different choice of frequency pairs, transformation can be achieved between other pairs of periodic orbits.

Conclusions

The main significance of the proposed control technique is that a given system can be driven to a desired periodic solution in excitation-amplitude–frequency-parameter plane. The advantage of the method is that direct attractor selection is possible meaning that the final state of the trajectory is not incidental (as in case of many control of multistability techniques). The disadvantage is that a detailed knowledge of the bifurcation structure of the periodic orbits is required to control the system confidently. It is worth mentioning that the technique is first identified in the previous paper of the authors [5]; however, only for a specific pair of periodic orbits and a generalisation was not discussed.

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