Modeling non-conventional vibrational modes of micro-plates in viscous fluids

Andre Loch Gesing*, Daniel Platz* and Ulrich Schmid*
**Institute of Sensor and Actuator Systems, TU Wien, Gußhausstraße 27, 1040 Vienna, Austria*

Summary. Micro-plates exhibit extraordinarily low losses in viscous fluids when vibrating in the non-conventional roof tile-shaped modes. However roof tile-shaped modes are commonly not considered for micro-sensors due to the lack of methods to predict these modes' dynamic response in viscous fluids. We developed a numerical method to calculate the spectral displacement of micro-plates oscillating in viscous fluids with which we can predict both conventional and non-conventional vibrational modes. The method is based on the Kirchhoff-Love plate equation, which we solve with a continuous-discontinuous approach to the Galerkin Method, with the method of fundamental solutions for the linearized Navier-Stokes equations. We show-case our method with the analysis of a silicon micro-plate immersed in water. With the spectral displacement curve it is straight forward to categorize peaks correspondent to beam bending modes, torsional modes and roof tile-shaped modes. Furthermore, we calculate the fluid flow field associated to each vibrating mode of the micro-plate. Our method thus provides a crucial understanding of the flow field around an oscillating micro-plate, which may enable novel device architectures for resonantly operated micro-sensors in viscous fluids.

Introduction

Commonly micro electro-mechanical systems (MEMS) devices exploit as key building block the Euler-Bernoulli (EB) flexural bending modes, or torsional modes, of narrow beams. These modes are chosen because methods to predict their dynamic response and thermal calibration in viscous fluids are well established and straightforwardly implemented [1]. Higher order bending modes of beams in viscous fluids are subject to high viscous losses. High order roof tile-shaped modes of micro-plates, on the other hand, exhibit extraordinarily low losses in viscous fluids [2]. However numerical methods for the prediction of the dynamic response of micro-plates in viscous fluids, and thus for roof tile shaped modes, are missing. Thus, a method for the prediction of the dynamic response of micro-plates in viscous fluids is of utmost importance to improve device performance even further.

Numerical Method

Micro-plates immersed in viscous fluids is a multi-scale problem, which means its solution via traditional numerical techniques - e.g. computational fluid dynamics and finite differences - is not straight forward. Our approach is to solve the Kirchhoff-Love (KL) plate equation combined with the method of fundamental solutions for the linearized Navier-Stokes (LNS) equations. KL plate theory is based on a fourth order partial differential equation (PDE), which can't be solved with out-of-the-box Finite Element Method (FEM). We solve the KL PDE with a continuous/discontinuous method called interior penalty (IP). IP method enables us to use Lagrange-type continuous elements (C0 continuous) while minimizing discontinuities in first order derivatives (obtaining thus C1 continuity) and weakly enforcing clamped and free-end boundary conditions. For comparison purposes we also implemented the IP method for the EB beam equation. To solve the fluid-structure interaction, we assume the non-slip condition at the plate's surface and assume that the fluid velocity in the longitudinal direction of the plate is small in comparison to the other components. With this simplification, there is a fundamental solution for the LNS that relates pressure at the plate surface and the plate's transverse displacement at each cross-section [3]. Evaluation of the fundamental solution at top and bottom surfaces of the plate results in a complex pressure jump over the plate which is linearly and non-locally dependent on the plate's transverse displacement.

Results

As an application example we consider a $300 \times 300 \times 5\ \mu\mathrm{m}^3$ silicon micro-plate immersed in water. The micro-plate is clamped on one side and free on the three others, and is excited by an external force at one of the free corners. Fig. 1 shows the displacement spectrum per unit excitation force ϕ/F of the free corner of the plate where the force is applied. Note that some of the peaks in the displacement spectrum are predicted by both KL and EB models. These peaks correspond, evidently, to flexural bending modes. The second peak in KL displacement spectrum corresponds to the first torsional mode. First beam bending mode and first torsional mode are shown in Figs. 2a and 2b, respectively. Roof tile modes in water occur at frequencies 237.4 kHz and 651.8 kHz and are shown in Figs. 2c and 2d. Naturally, note that these modes are predicted only with the KL method. Once the plate spectral displacement is calculated, we can predict the velocity field at any point of the fluid domain by applying once again the fundamental solutions for the LNS. The results of this procedure are shown in Fig. 3, where the velocity field is shown for the plate's vibrational modes shown in Fig. 2. Velocity field associated with the first bending mode exhibits two vortices at the edges of the plate's cross section with opposite directions, with the fluid near the middle being dominated by a normal component. For the torsional mode we see the vortices with opposite directions at the edges, and the fluid moves across the plate's surface dominated by a tangential component. For roof tile modes the fluid moves back and forth across the plate's surface with a pattern that varies with the number of nodal lines of the vibrating mode. Note also that for roof tile modes the velocity field can reach higher velocities near the middle of the plate than at its edges.

Conclusions

We showed that with the KL method we can model non-conventional modes of micro-plates in viscous fluids besides the traditional bending and torsional modes. What is more, we are able to calculate the fluid flow associated to each vibrational mode, which in turn enable novel strategies to further decrease energy losses to the fluid. Future developments include implement 3D fundamental solutions for the LNS and comparison to experimental data.

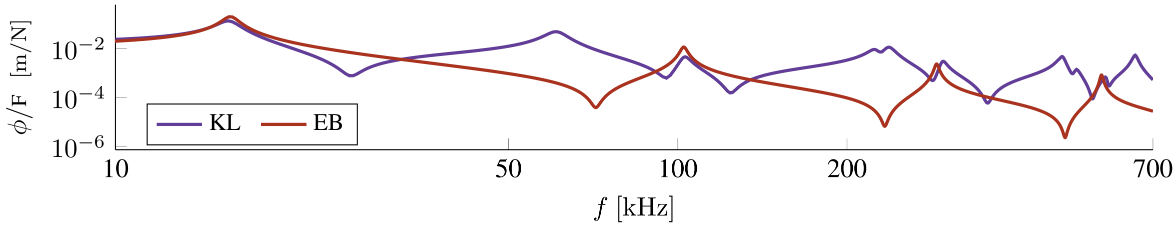

Figure 1: Displacement of a corner of the plate per unit excitation force calculated with KL and EB methods.

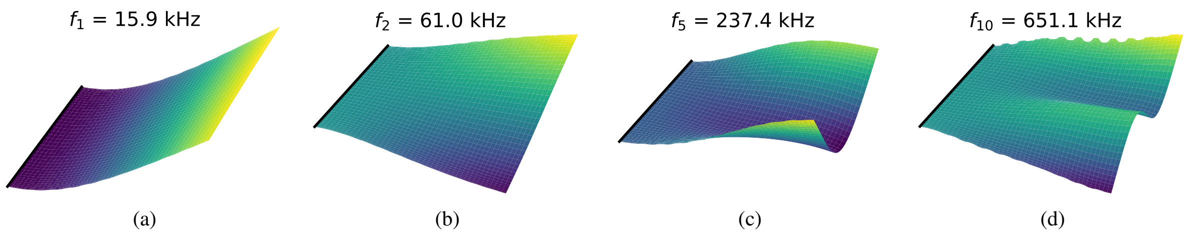

Figure 2: (a) First flexural, (b) first torsional, (c) first and (d) second roof tile-shaped vibrating modes.

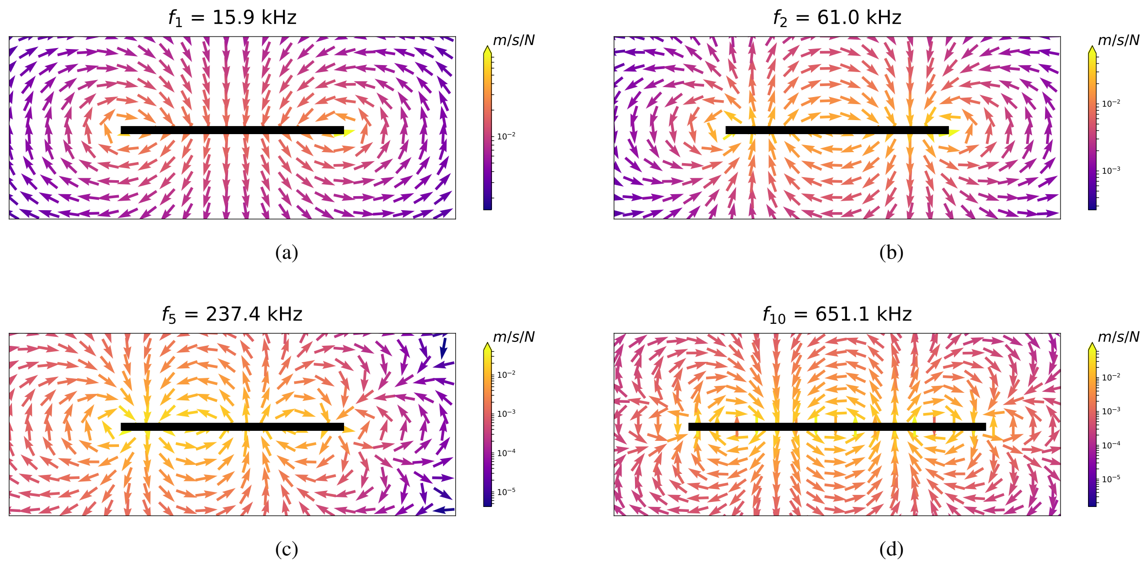

Figure 3: Fluid flow associated to the plate's different mode shapes.

References

[1] J. E. Sader, "Frequency response of cantilever beams immersed in viscous fluids with applications to the atomic force microscope," *Journal of Applied Physics*, vol. 84, no. 1, pp. 64–76, 1998.

[2] M. Kucera, E. Wistrela, G. Pfusterschmied, V. R. Díez, J. L. S. Rojas, J. Schalko, A. Bittner, and U. Schmid, "Characterisation of multi roof tile-shaped out-of-plane vibrational modes in aluminium-nitride-actuated self-sensing micro-resonators in liquid media," *Applied Physics Letters*, vol. 107, no. 5, pp. 1–5, 2015.

[3] E. O. Tuck, "Calculation of unsteady flows due to small motions of cylinders in a viscous fluid," *Journal of Engineering Mathematics*, vol. 3, no. 1, pp. 29–44, 1969.