Utilizing Noise to Manipulate Energy Localization in a Circular Oscillator Array

Gizem Dilber Acar*, Abdulrahman Alofi* and Balakumar Balachandran *
*Department of Mechanical Engineering, University of Maryland, College Park, 20742, USA

Summary. In this work, energy localization in an array of Duffing oscillators with periodic boundary conditions is studied. Each oscillator in this array can exhibit multi-stable behavior, and the localized mode of the forced array exists in a certain frequency range. For a specific excitation frequency, Gaussian noise is applied in addition to the harmonic forcing to move the system response from the localized mode to a low-amplitude mode. Through this study, the authors shed light on how noise can be used to design the desired dynamics of coupled oscillator arrays, such as a circular array of rotor blades.

Introduction

Coupled oscillator arrays represent numerous mechanical systems including multi-bladed rotors and micro- electro- mechanical resonators [1]. Understanding nonlinear behavior of oscillator arrays is important for designing such systems. Energy localization is a nonlinear phenomenon that can be observed in oscillator arrays, wherein the system's energy is spatially focused in one or more oscillators. Although spatial energy localization can happen in discrete systems due to imperfections, it can be observed in homogeneous discrete systems due to nonlinearity [2]. Depending on the application, spatial localization of energy can be desirable or harmful.

Cyclically symmetric, discrete systems can exhibit localization behavior when the coupling between the discrete elements is weak [3]. In this study, the responses of an array subjected to a harmonic forcing and harmonic forcing with noise are numerically examined. The system exhibits multi-stable behavior in a particular frequency range, and depending on the initial conditions, one of these responses can correspond to energy localization. The array has a circular configuration (i.e., periodic boundary conditions), and each oscillator is coupled with two neighbors. Furthermore, energy can flow through the boundaries without interruptions. Here, the system is excited with a harmonic forcing so as to excite a localized mode. Then, Gaussian noise is added to the harmonic forcing in order to steer the system to a non-localized response. By using the numerical studies, the authors show that the Gaussian input can be used to move the system from the localized mode to a low-amplitude mode, wherein all the oscillator move with the same amplitude, and the energy is uniformly distributed across the array. For different noise intensities, the authors explore the possibility of using noise to destroy energy localization.

Numerical Experiments

A circular array of six identical hardening Duffing oscillators are considered, wherein each oscillator has associated linear stiffness, cubic stiffness, and linear damping. Each oscillator is coupled with the neighbors through linear springs, as shown in Figure 1a. The system is cyclically symmetric with periodic boundary conditions. The localized mode of the system is found using the anti-continuous limit method [4], according to which the localized mode is first found for the system with zero coupling stiffness. Then, by gradually increasing the coupling in small steps, and using the numerical shooting method at each step, the localized mode of the coupled system is found. The array is excited with a harmonic forcing that can induce energy localization in the system. A representative amplitude profile of the localized mode is shown in Figure 1b. The localized mode is symmetric about the high-amplitude oscillator, and each oscillator moves out-of-phase with its two neighbors. In this analysis, the authors chose the 3^{rd} oscillator for the localization. It is noted that with a different choice of initial conditions, the system energy can be localized in another oscillator. Although energy localization can be observed in systems with smaller numbers of degrees of freedom, the authors analyzed an array with six oscillators to show that the amplitude of oscillations varies drastically around the localization, and it decreases as one moves away from the high-amplitude oscillator. For arrays with larger numbers of oscillators, the amplitude profile may become more uniform across the oscillators that are away from the high-amplitude oscillator.

In order to analyze the system behavior under noise, a Gaussian input is added to the harmonic forcing, as shown below.

$$\ddot{x}_n + c\dot{x}_n + k_1 x_n + k_3 x_n^3 + k_c(2x_n - x_{n-1} - x_{n+1}) = F_0 \cos(\omega t) + \sigma \dot{W}(t), \tag{1}$$

for $n = 1, \ldots, N$, where the periodic boundary conditions imply that the N + 1 oscillator coincides with the first oscillator. The incremental noise is represented with $\sigma \dot{W}(t)$. In this study, each oscillator is excited with the same forcing function to avoid inducing asymmetry into the system through forcing. The system equations with noise are put into Langevin form, and the Euler Maruyama method is used to numerically integrate the noise-influenced system equations [5]. For three different noise intensity levels, the obtained energy distribution in the oscillator array is shown in Figure 2. For each chosen noise intensity, the energy distribution plots are provided by using an average of the responses of the oscillator array to 400 different noise vectors. Although depending on the noise vector, it might take longer or shorter for the noise to suppress localization, the averaged dynamics show that for higher noise intensities ($\sigma = 0.01$ and $\sigma = 0.006$), the Gaussian input can destroy the energy localization, and push the system to a state with uniform energy distribution. However, for a smaller noise intensity ($\sigma = 0.002$), one is not able to use the noise to the unison amplitude mode, and the localization persists.

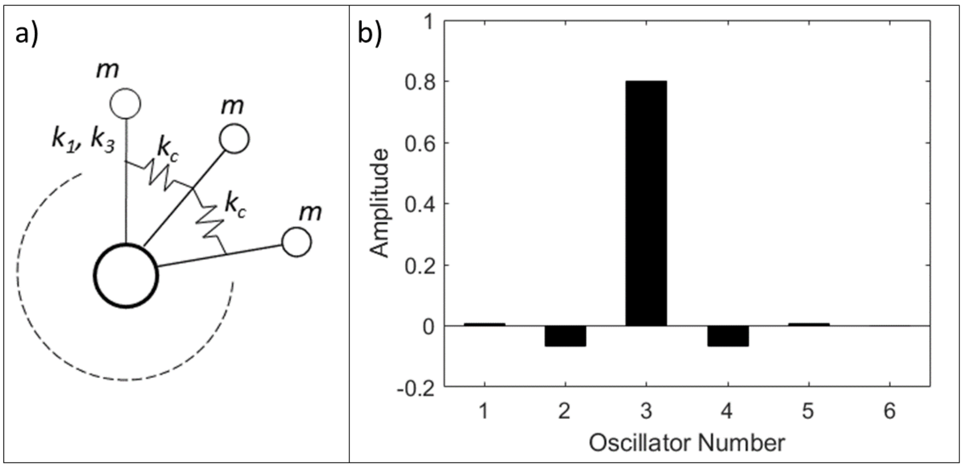

Figure 1: a) Coupled circular Duffing oscillator array with periodic boundary conditions. b) Amplitude profile of the localized mode: The energy is localized in the 3^{rd} oscillator, and the amplitude profile is symmetric about the oscillator with the highest amplitude.

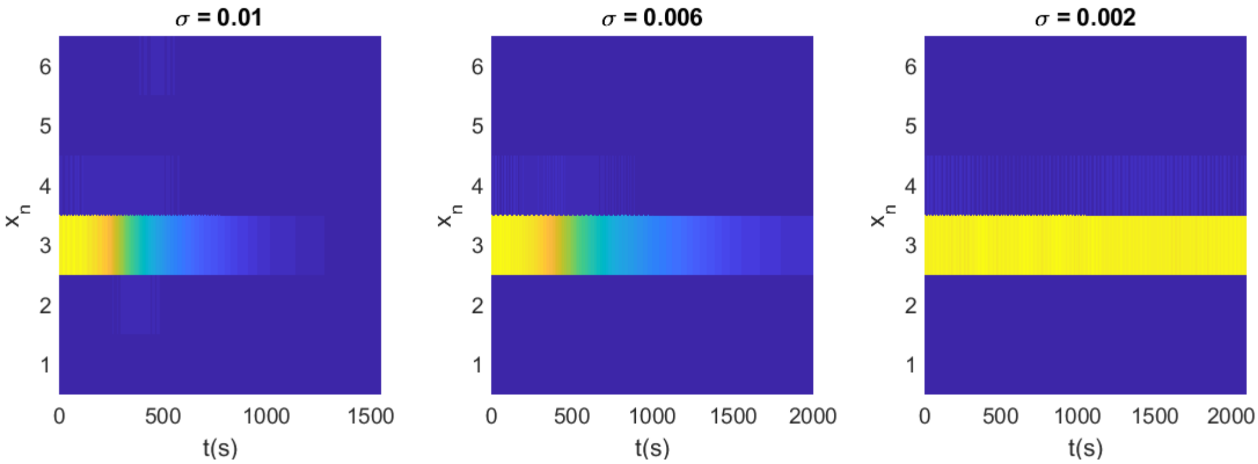

Figure 2: Energy distribution in a circular Duffing oscillator array averaged over 400 simulations. In all three cases, the energy is localized in the 3^{rd} oscillator in the beginning. Addition of noise with intensity $\sigma = 0.01$ and $\sigma = 0.006$ can suppress the energy localization, and lead to a uniform energy distribution in the array. However, a lower noise intensity ($\sigma = 0.002$) is found to be inadequate for pushing the system to the unison-amplitude mode.

Conclusions

In this study, energy localization in a circular array of hardening Duffing oscillators with weak, linear coupling was investigated. The system was first excited with a harmonic forcing frequency, for which the array can oscillate in a localized mode. Then, various levels of Gaussian noise were applied along with the harmonic forcing, to explore the effects of noise on energy localization. Through numerical studies, it was found that above a certain noise level, the localization can be suppressed. The findings may provide a basis for suppressing energy localization in arrays of turbine blades. The studies will be extended to larger systems in order to explore the effects of number of oscillators on the noise required to drive the system between solutions with different energy distributions.

Acknowledgment

The authors are grateful for the support received from the U.S. National Science Foundation, through grant CMMI-1760366.

References

[1] Dick A. J., Balachandran B., Mote C. D. (2008) Intrinsic localized modes in microresonator arrays and their relationship to nonlinear vibration modes. *Nonlinear Dynamics* **54.1-2**: 13-29.

[2] Sievers A.J., Takeno S. (1988) Intrinsic localized modes in anharmonic crystals. *Physical Review Letters* **61.8**: 970.

[3] Vakais Alexander F., Cetinkaya C. (1993) Mode localization in a class of multidegree-of-freedom nonlinear systems with cyclic symmetry. *SIAM Journal on Applied Mathematics* **53.1**: 265-282.

[4] Marin J. L., Aubry S. (1996) Breathers in nonlinear lattices: numerical calculation from the anticontinuous limit. *Nonlinearity* **9.6**: 1501.

[5] Higham D. J. (2010) An algorithmic introduction to numerical simulation of stochastic differential equations. *SIAM review* **43.3**: 525-546.