Cracking Down on Criminals: A Mathematical Model Expoloring Strategies for Curbing Criminal Behaviour

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Summary. Criminal behaviour is a rapidly growing challenge internationally. Policy makers are usually charged with developing strategies to control its spread - while limited by financial constraints. With the use of a model adapted from epidemiology, control strategies in the form of time dependent prevention and treatment efforts to curtail the spread of criminal behaviour may be evaluated. The first control strategy applied encourages potential criminals away from a life of crime while the second strategy targets criminals. We find that a combination of strategies leads to the biggest reduction in the number of criminals and of potential criminals. To be effective, strategies based on single controls require the implementation of more intensive efforts at the start of the control effort.

Extended Abstract

One of the greatest social challenges plaguing many countries nowadays is that of criminal behaviour and its ensuing prevention and reduction. Violent crime can have a negative effect on a countrys economy in a variety of ways. These include reduced economic growth, a decline in tourism, increased emigration in conjunction with a resulting brain drain, a reluctance to invest locally by foreign investors and a general feeling of fear and insecurity by the population. This is compounded when limited resources are available to policy makers. Especially in such instances, whatever measures are used must be cost effective.
An innovative approach to tackling crime that is rapidly gaining popularity is to consider criminal behaviour as an infectious disease and then to use a public health approach to mitigate its spread [1, 2]. This approach may also be used when designing strategies to prevent or disrupt the contagion process, which may then be tested using mathematical models from epidemiology - compartmental models.
Optimal control theory has been implemented to study strategies for the treatment of many diseases including waterborne diseases, HIV, Ebola, Dengue and Pandemic Influenza. In recent times [3], modelling of behaviour using an infectious disease approach has been done for fanatical and violent ideology, violent crime and burglary and gang membership. Our paper deals with the application of optimal control to a dynamic model of criminal behaviour treated as an infectious disease. Our aim is to find the best strategy in terms of combined efforts of prevention and treatment that would minimize the total number of criminals and its cost.

Model Formulation
The population under consideration $N(t)$ consists of people who are "at risk"for engaging in or have engaged in criminal behaviour. N is divided into four disjoint compartments/classes based on status with respect to criminal behavior. These are:

P : Members of the population who are susceptible to criminal behaviour - Potential Criminals.

C : Members of the population who are engaged in criminal behaviour - Criminals.

J : Members of the population who are in prison or in a juvenile delinquency centre.

R : Members of the population who were in an prevention or treatment program and have permanently left life of crime - Recovered people.

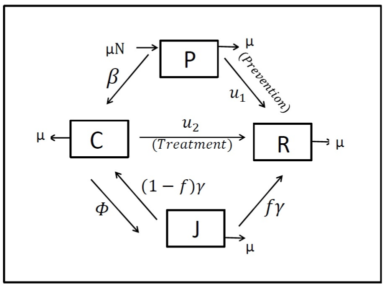

Figure 1: Model Diagram

We consider two separate control programs $u_1(t)$ and $u_2(t)$ applied to the P and C subgroups with the goal of "recovery". These represent programs or strategies which encourage positive behaviour change and include programs designed by the government as well as outreach programs by social service groups, neighbourhood organizations, and the faith community.

The first control strategy $u_1(t)$ is applied to potential criminals P who may become criminals by interaction with criminals C at the rate $\frac{\beta CP}{N}$. However, since a prevention control scheme is applied to the P class, $u_1(t)P$ individuals progress to the recovered class R. Individuals in the criminal class may enter a treatment control program at the rate $u_2(t)$ or go to jail at the rate ϕ. On release from jail, a fraction $(1-f)$ re-join the criminal class and the remainder enters the recovered class at a rate of $f\gamma$ where γ^{-1} represents the time spent in jail. For ease of analysis purposes, we assume a constant size population N with an entry and death rate into the population given by μ. The rate of entry into the system is proportional to the population size and is given by μN. Figure 1 shows the structure of flows within the model which is described by the following set of nonlinear differential equations:

$$P' = \mu N - \frac{\beta CP}{N} - u_1(t)P - \mu P \tag{1}$$

$$C' = \frac{\beta CP}{N} + (1-f)\gamma J - u_2(t)C - \phi C - \mu C \tag{2}$$

$$J' = \phi C - \gamma J - \mu J \tag{3}$$

$$R' = u_1(t)P + u_2(t)C + f\gamma J - \mu R \tag{4}$$

$$N = P + C + J + R \tag{5}$$

Model Analysis – Equilibria and Stability

We study the existence and stability behaviour of the system at its equilibrium points. Two possible equilibrium states are found – the criminal-free equilibrium where the system only consists of potential criminals and the coexistence equilibrium. Coexistence (or endemic) equilibrium points are steady-state solutions where the disease “criminality”persists in the population.

The issue of whether or not a disease can invade a host population and persist or remain endemic involves the introduction of a threshold known as the basic reproductive number R_0. In our model, R_0 represents the average number of potential criminals who are recruited to the criminal class c. If $R_0 < 1$, then the criminal-free equilibrium is locally asymptotically stable. If $R_0 > 1$, then the criminal-free equilibrium is unstable, and the introduction of a criminal will result in an outbreak. In the early stages of a crime outbreak, R_0 is the key quantity of interest, and our goal is to identify mitigation strategies to reduce it below the threshold $R_0 = 1$.

Formulation of the Optimal Control Problem

Our first control strategy ($0 \leq u_1(t) \leq 1$) includes programs and practices that target individuals who have an elevated risk for becoming criminals. The second control strategy ($0 \leq u_2(t) \leq 1$) includes programs to treat criminals. Our goal is to minimize the number of criminals while at the same time minimizing the cost of controls $u_1(t)$, $u_2(t)$ over a time period T given the initial population sizes of all three classes $p(0)$, $c(0)$ and $j(0)$. Thus, we are seeking the optimal control pair $(u_1^*(t), u_2^*(t))$ so that

$$J(u_1^*, u_2^*) = min\{J(u_1, u_2) : (u_1, u_2) \in U\} \tag{6}$$

where the Lebesgue measurable control set U is defined as

$$U = \{u_1(t), u_2(t) : 0 \leq u_1 \leq 1, 0 \leq u_2 \leq 1, t \in [0, T]\} \tag{7}$$

subject to the model equations.

The objective functional is defined as

$$J(u_1, u_2) = \int_0^T \left[Kc(t) + \frac{B_1}{2}u_1^2 + \frac{B_2}{2}u_2^2 \right] dt \tag{8}$$

where B_1 and B_2 are the relative weights attached to the cost or effort required (human effort, material resources, infrastructural resources etc.) to implement each of the control measures. K measures the comparative importance of the criminal burden relative to the control costs. Pontryagin’s maximum principle [4] is used to solve this optimal control problem numerically using the backward-forward sweep method [5, 6] with the initial conditions given in [7] and reasonable estimates for the model parameters.

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