Methods for decreasing order and dimension in mechanics of solids

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Summary: The paper gives further development of methods for reducing a system of partial differential equations (PDEs) to a system of ordinary differential equations (ODEs). Iterative methods are proposed which at each loading step for static problems and at each time step employ suitable approximating functions. The Bubnov-Galerkin method was considered as a basis for further consideration. The effectiveness of the developed methods were demonstrated by solving the problems of the theory of plates and shells. The proposed approaches allow us to solve a wide class of problems in linear/nonlinear formulation for both full-sized and nanoscale structures. The problems were solved taking into account the temperature field, geometric and physical nonlinearities, contact interaction between structures, as well as the Casimir and Van der Waals effects. The convergence theorems of solutions for some iterative methods were proved. Software packages for solving problems of statics/dynamics were developed. The exact solutions regarding plates were given taking into account the nanoscale parameter and constant load. Then exact solution was compared with solutions obtained by two iterative methods.

Introduction

Such problems are usually solved using the following methods: finite difference methods (FDM), finite element method (FEM) and in result a system of linear or nonlinear algebraic equations is obtained, which bounds application of these methods.

Mathematical models and solution methods

Iterative methods for reducing PDEs to a system of ODEs were constructed. These methods are based on the Bubnov-Galerkin method, and the method of Kantorovich–Vlasov (MKV) [1] . The latter method, by its ideology, match the Fourier method (MF) based on the variables separation and the Bubnov-Galerkin method (MBG), which gave impetus to a number of modifications (Fig. 1) including the following modifications: the Vaindiner method (MV)[2], the variational iteration method (MVI) [3] or extended Kantorovich method (EKM) [4], the Agranovsky – Baglai – Smirnov method (MABS) [5-6] and their combinations [7-10]. These articles provide evidence of convergence and a comparative analysis of the results can be applicable for full-sized structures. The method of variational iterations (MVI) eliminates the need to construct a system of approximating functions in the procedure of employment of the Bubnov-Galerkin method. The functions initially specified in an arbitrary way (obviously satisfying certain well-known smoothness conditions) were refined in the process of calculations by MVI based on the solutions of the original system of PDEs. The resulting system of functions was given and the Bubny-Galerkin method was used to obtain a system of ODEs with respect to another variable. This iterative process continues to obtain a solution with a given accuracy. In this paper, these methods are implemented to solve a class of problems in the mechanics of a continuous deformable medium. The nanostructures are described using the modified couple stress theory [11] and the classical theory of shallow shells. Contact interaction is investigated using the theory of Kantor. Physical nonlinearity is taken into account according to the deformation theory. The temperature field is determined from the solution of a three-dimensional heat PDE. The relationship between the strain fields and temperature is not taken into account

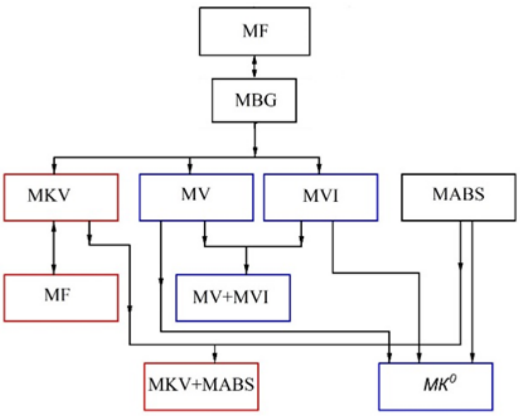

Fig.1 Interrelation of the Bubnov-Galerkin method, the Fourier method, the method of Kantorovich-Vlasov and their modifications

The effects of Casimir [12], Van der Waals are taken into account as well as a geometric non-linearity based on the von Kármán theory. The governing partial differential equations, boundary and initial conditions are obtained from the Hamilton principle and the calculus of variations. For a number of approaches (Fig. 1), theorems were formulated and proved for the existence and uniqueness of solution for the equations of nanostructures of rectangular shells in terms of geometric nonlinearity.

Results

As a numerical example, the results of a study of the static problem of a square in terms of a nanoplate are presented. The load is uniformly distributed over the plate area. Plate material is isotropic and elastic with Poisson's ratio ν=0.3. The solution is obtained for two types of boundary conditions. The problems are solved in a linear formulation for a size dependent parameter γ=0 and γ=0.5. Exact solutions have been obtained.

Table 1.

Boundary condition	Exact solution	γ	MVI	MVI+MABS
Hinge	0.2028	γ=0	0.2030	0.2028
Rigid jamming	0.0661		0.0653	0.0661
Hinge	0.1064	γ=0.5	0.1073	0.1065
Rigid	0.0381		0.0376	0.0383

The solutions are obtained by a combination of MVI and MVI + MABS. These solutions practically coincide with the exact solution of the considered problems. This indicates the effectiveness of these methods. These two approaches have their advantages: there was no need to build an initial approximation that satisfies the given boundary conditions of the problem. The methods (MV) and (MV+MABS) have the same property. To analyze the problems of nonlinear dynamics, the methods of the theory of differential equations are used including analyses of signals, phase portraits, Fourier power spectra and wavelets. In addition, the Lyapunov exponents are numerically estimated using the Wolf, Kantz, and Rosenstein methods. Static problems are obtained from dynamic solutions using the set up method.

Remark. The approach proposed in Fig. 1 applies to elliptic equations. It is possible to extend it to linear and nonlinear hyperbolic equations. Then the solution can be represented as a product of three functions, each of which depends on one variable *t,x,y*. В Reducing the PDE's to ODE's regarding *t* at each time step, it was proposed to use the methods described in Fig. 1.

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