Bifurcation analysis at a degenerate parameter point of a non-collocated force control model

Li Zhang[*], Huailei Wang[*] and Gabor Stepan[**]
[*]*College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, China*
[**]*Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, Hungary*

Summary. Normal form analysis is carried out at a degenerate parameter point of the low degree-of-freedom non-collocated delayed force control model. It is shown that around the degenerate point where both the derivative of the real part of the eigenvalues and the first Lyapunov coefficient are zero, neutral stable periodic orbits arise and a specific transcritical bifurcation of limit cycles takes place.

Introduction

Force control is a relevant task of human motion control when an operator touches an object or the environment, and it is often studied as important part of the efforts for understanding human behavior. Due to the processing time of the sensory signals in the neural system, and also to the reaction time of the muscles, delay is one of the essential parameters in force control. This study will present bifurcation analysis at a degenerate parameter point of a non-collocated force control model in the presence of human reaction time delays.

Mechanical model

A basic model of delayed force control with non-collocated force sensor configuration is considered. A block of mass m is in contact with the rigid environment via a spring of stiffness k_1 along a horizontal axis as shown in Figure 1. The contact force can be obtained by a force sensor, which is a serially connected spring of large stiffness $k_2(\gg k_1)$ located at the contact point to the rigid environment. The sensed signal is fed back to the control force Q of the human actuation. The governing equation takes the following form:

$$\begin{aligned} m\ddot{q}_1 &= Q - k_1(q_1 - q_2) \\ 0 &= k_1(q_1 - q_2) - k_2 q_2 \end{aligned} \tag{1}$$

where q_1 and q_2 are the absolute positions of the block and the end point of the spring that detects the force, respectively. With a simple control strategy and with saturation of the control force into consideration, the actual control force Q at time instant $\tilde{\tau}$ is given by

$$Q(\tilde{t}) = -F_S \tanh\left(\frac{1}{F_S} P(k_2 q_2(\tilde{t} - \tau) - F_d)\right) + k_2 q_2(\tilde{t} - \tau) \tag{2}$$

where P is the feedback gain, τ is the reaction time of the actuator, and F_S can describe the level of the actuator force saturation. By shifting $q_1(t)$ by the equilibrium position q_{10} of the block m, i.e., by introducing the coordinate $\tilde{x}(t) = q_1(t) - q_{10}$, using the assumption $k_2 \gg k_1$, and introducing the dimensionless coordinates $x = \tilde{x}/(F_S/k_1)$ and $t = \tilde{t}/\tau$, the Newtonian equation (1) is transformed to:

$$\ddot{x}(t) + (\omega_n\tau)^2 x(t) = (\omega_n\tau)^2 x(t-1) - (\omega_n\tau)^2 \tanh(Px(t-1)) \quad . \tag{3}$$

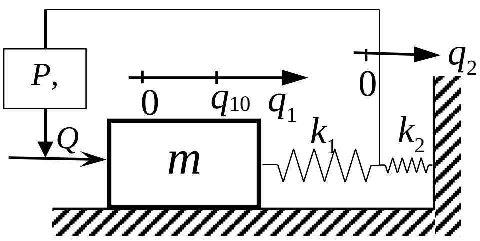

Figure 1: A basic model of non-collocated delayed force control

Linear Analysis

By means of analyzing the characteristic equation of the linear part of Equation (3), the stability chart in the parameter plane $(P, \omega_n\tau)$ is obtained as shown in Figure 2. The stability boundaries and regions are determined according to the number of characteristic roots with positive real parts.

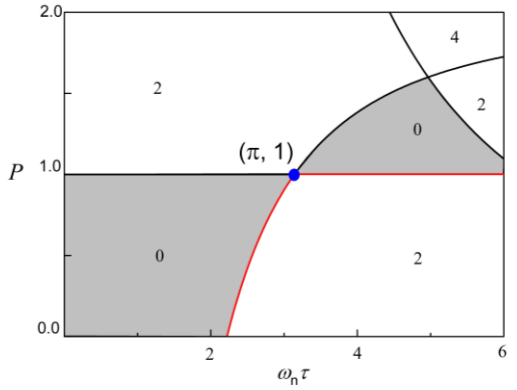

Figure 2: Stability chart in the parameter plane $(P, \omega_n\tau)$. Shaded regions refer to stability. Numbers indicate the number of characteristic roots with positive real parts. Black stability boundaries refer to supercritical Hopf bifurcations and red ones refer to subcritical Hopf bifurcations. The blue point at (π, 1) is the degenerate parameter point.

The 5th order Normal Form at the degenerate parameter point

According to [1], the black stability boundaries refer to supercritical Hopf bifurcations and the red stability boundaries refer to subcritical Hopf bifurcations in Figure 2. At the blue point (π, 1), the sense of Hopf bifurcations changes and the stability of the equilibrium swaps, which indicates that both the first Lyapunov coefficient and the derivative of the real part of the eigenvalues at the critical parameters are zero. To study this special degenerate parameter point, normal form up to the 5th order is carried out via symbolic calculation [2]. Let $P = 1 + \mu$, then the normal form reads

$$\dot{y}_1 = i\omega_c y + S_1 y\mu + S_2 y\mu^2 + \Delta_1 y^2\bar{y} + S_3 y^2\bar{y}\mu + \Delta_2 y^3\bar{y}^2 \tag{4}$$

where $\mathrm{Re}(S_1) = 0$, $\mathrm{Re}(\Delta_1) = 0$, $\mathrm{Re}(S_2) = \pi^2/4$, $\mathrm{Re}(S_3) = -\pi^2/2$ and $\mathrm{Re}(\Delta_2) = \pi^2/4$. Therefore, the neutral stable bifurcated periodic vibration has amplitude ρ

$$\rho = 2\sqrt{1-\mu} \tag{5}$$

as shown in Figure 3.

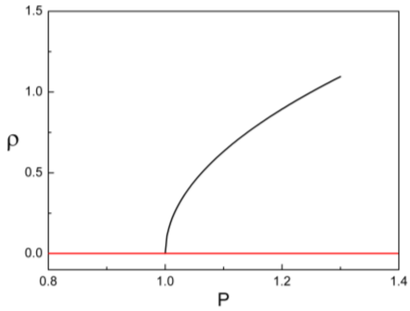

Figure 3: Bifurcation diagram of periodic motions with respect to feedback gain P at $\omega_n\tau = \pi$. Red line refers to unstable equilibrium.

Conclusion

The normal form up to the fifth order is obtained at the degenerate parameter point where both the derivative of the real part of the critical eigenvalues and the first Lyapunov coefficient at the critical parameters are zero for the simple non-collocated force control model. The nonlinear analysis shows that neutral stable periodic orbits arise around this point. This is not a standard fold bifurcation, but transcritical bifurcation of limit cycles, which happens around this degenerate parameter point.

Acknowledgement This research was supported by the Hungarian-Chinese Bilateral Scientific and Technological Cooperation Fund under grant no. 2018-2.1.14-TÉT-CN-2018-00008, the National Natural Science Foundation of China under Grants No. 11772151.

References

[1] Zhang L., Stepan G. (2019) Bifurcations in basic models of delayed force control. Nonlinear Dyn. https://doi.org/10.1007/s11071-019-05058-7.
[2] Zhang L., Wang H.L., Hu H.Y. (2012) Symbolic Computation of Normal Form for Hopf Bifurcation in a Neutral Delay Differential Equation and an Application to a Controlled Crane. Nonlinear Dyn. 70: 463-473.