Swing Oscillations Generated by Sitting Human

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Summary. Oscillations pumped by the human sitting on the swing are modeled. The model is represented by a three-link mechanism. Control torques are applied in the knee- and hip- joints. The control strategy is proposed that allows generation and maintaining of oscillations with large amplitude and also rotation around the suspension point. Control torques are designed as autonomous feedbacks depending on the angle and the angular speed of rotation of the swing. Numerical simulation is performed. The anthropomorphism of the proposed control is illustrated.

Introduction

Pumping of a swing is systematically discussed in the literature as a classical example of forced oscillations. This paper deals with the case of pumping a rigid swing by a sitting person. The paper provides the following novel results: it is shown that the anthropomorphic strategy of pumping is quasi-optimal one, the control strategy is proposed that can pump the swing to stationary rotation (not only to oscillations), the numerical simulation is performed for the three-link model of the system while usually only oscillations of a double-link model are discussed (see, for example, [1-5]).

Statement of the Problem

Three-link planar hinge mechanism models a human pumping a swing (see Fig. 1a). Link *VP* models a body, link *PK* – both thighs, and *KF* – both shins. Link *PK* is rigidly joined to the rod that corresponds to the swing. The rod is pivotally joined to the suspension point *O* that is the axis of rotation of the swing. The viscous friction torque with coefficient *c* acts at this axis. The angle of rotation of the swing is denoted as φ. One inter-link hinge *K* models two knee-joints, second hinge *P* – two hip-joints. Two limited control torques are applied at these two inter-link joints *K* and *P*. Thus, the system has three degrees of freedom, but only two control torques are applied. So, this system is under-actuated one. Angles α in joint *K* and β in joint *P* can be changed in limits $\alpha_{min} \le \alpha \le \alpha_{max}$ and $\beta_{min} \le \beta \le \beta_{max}$. Here $\alpha_{min/max}$, $\beta_{min/max}$ are given constants. The limitation upon these angles is modeled via relatively strong single-side spiral springs in joints *K* and *P* which practically ensures that angles α and β do not leave the prescribed domain. These springs model ligaments and tendons.

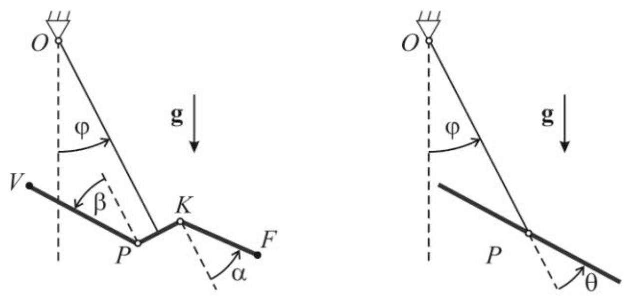

Figure 1: The schemes of the three-link and simplified double-link mechanisms

The goal of the control is to pump the swing as fast as possible and to the largest amplitude taking into account the restrictions imposed on the control. In this paper, the solution that is "close to optimal" is proposed.

Design of the control and simulation

As a preliminary step the quasi-optimal control is designed for the simplified double-link model in which a human is substituted by a single rigid body (see Fig. 1b) and the limited angle θ is a control input. This control strategy is represented by a relay-type function $\theta^*(\varphi,\dot\varphi)$ that ensures pumping the swing to the large amplitude oscillations or even to stationary rotations (depending on parameters $\theta_{min/max}$ and friction coefficient *c* in the suspension joint):

$$\theta^*(\varphi,\dot\varphi) = \begin{cases} \theta_{max}, & \text{if} \quad \dot\varphi\cos\varphi \ge 0, \\ \theta_{min}, & \text{if} \quad \dot\varphi\cos\varphi < 0. \end{cases}$$

Function $\theta^*(\varphi,\dot{\varphi})$ is used as the program function in the initial problem for the three-link mechanism (with $\theta_{min/max}$ equal to $\alpha_{min/max}$ or $\beta_{min/max}$ correspondingly). Control torques are designed as combinations of linear feedbacks with respect to the differences between the current and program angles and their derivatives. Efficiency of this control strategy is illustrated by numerical simulation with parameters of the model similar to parameters of the human. This simulation shows that if friction coefficient *c* is rather large, then oscillations of the swing tend to a stationary oscillatory mode corresponding to a cycle in the configuration space (see Fig. 2). But if friction coefficient *c* is sufficiently small, then the swing is pumped to a stationary rotation corresponding to the cycle in the space $(\varphi \bmod 2\pi, \alpha, \beta)$. These cycles are attracting ones.

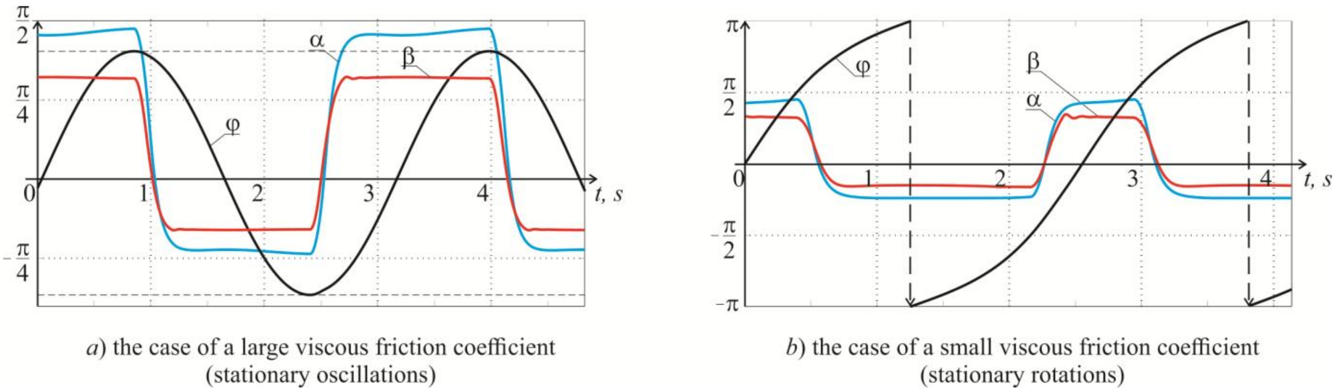

Figure 2: Illustration of stationary oscillations and rotations of the swing

Conclusion

The designed control allows maintaining oscillations in wide range of amplitudes as well as rotational motion with a constant period depending on the restrictions upon angles α, β and on the viscous friction coefficient *c*.

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