Nonlinear Modal Testing of Structures with Nonlinear Dissipation

Maren Scheel* and Malte Krack*
**University of Stuttgart, Stuttgart, Germany*

Summary. The dynamics of many technical structures are characterized by both conservative and non-conservative nonlinear forces, which are often challenging to model. Alternatively, a nonlinear model can be identified from experimental data, yielding amplitude-dependent modal properties, which can also serve for model validation or model updating purposes. In this contribution, we present a nonlinear modal testing approach that allows for extracting modal properties for systems with nonlinear damping caused by various types of nonlinearities. Models based on these modal properties describe well the systems' steady-state dynamics around an isolated resonance.

Introduction

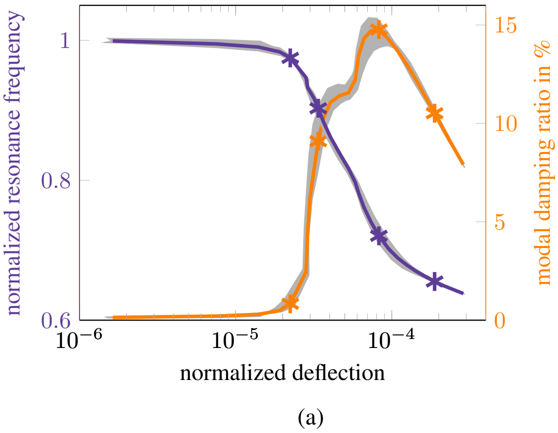
Technical structures are usually assembled by several parts, connected through joints. These joints are a common source of nonlinear, dissipative forces due to dry friction. To accurately predict vibrations of such structures, the nonlinear force-deflection relations have to be modeled. In particular, physical processes causing damping are in many cases inherently nonlinear and are properly described only by nonlinear hysteresis models. Therefore, experiments are crucial in order to estimate parameters for the purpose of model updating or to validate numerical models. To this end, experimental methods are needed, which are suited for structures under the influence of conservative and non-conservative nonlinear restoring forces. Different approaches for nonlinear system identification have been suggested in the last years [1]. One approach is to employ the concept of nonlinear vibration modes. If nonlinear forces play a significant role, one set of linear modal properties is of limited use, and nonlinear, i.e. amplitude-dependent, modal properties are needed. These can, for example, be extracted by analyzing the freely decaying response. If the structure is subjected to high damping, e.g. due to joints, the motion decays quickly, which is challenging from a signal analysis perspective. In such cases, using force appropriation to excite steady-state motion is preferable because it allows for a fine frequency resolution and straight-forward signal analysis. Force appropriation can be ensured using controlled excitation, for example with a phase-locked loop [2, 3] or control-based continuation [4]. In this contribution, we successfully apply a nonlinear modal testing method that utilizes steady-state force appropriation to experimentally extract amplitude-dependent modal properties of structures with nonlinear dissipation. The proposed approach is robust against noise and time efficient, which reduces the risk of damaging the specimen. Moreover, the method does not require any prior knowledge on the type or location of the nonlinearity. Using controlled excitation or a control-free alternative, the method overcomes problems related to stability loss of periodic responses, such as jumps, which often hamper the applicability of common frequency response testing (frequency stepping or sweeping). Once the modal properties are identified, it is no longer necessary to measure the frequency response around a resonance, since this can be accurately predicted by assuming that the vibration energy is confined to a single nonlinear mode. The applicability of the nonlinear modal oscillator is, however, limited to single-frequency, near-resonant forcing.

Nonlinear Modal Testing Method

The proposed nonlinear modal testing method is based on the extended periodic motion concept [5], which was introduced to study periodic motion of damped systems, close to a primary resonance. A nonlinear mode according to this concept is a periodic motion of an autonomous system. Periodicity is enforced by an artificial negative damping term that compensates the dissipated energy over one cycle of vibration. In the absence of strong modal interactions, a single nonlinear mode according to this definition accurately describes the steady-state forced response near resonance.
In an experiment, the negative damping term can be approximated by external forcing. We have shown that applying a force at only one location is sufficient for many structures. The excitation force must be in local phase resonance with respect to the fundamental harmonic of the response [6], which is ensured using a phase-locked loop controller, adapting the excitation frequency until local phase resonance is achieved. Alternatively, the system's velocity is scaled and fed back as excitation signal [7]. Varying the excitation amplitude, the amplitude-dependent resonance frequency (i.e. the backbone) is measured. From the Fourier transform of the measured steady-state time signals, the resonance frequency as well as the deflection shapes are extracted, and the modal damping ratio is obtained with a power balance [6].

Experimental Nonlinear Modal Analysis of a Strongly Friction-Damped Beam

The proposed nonlinear modal testing method has been applied to specimens with different sources of nonlinear dissipation such as bolted joints but also systems with nonlinear stiffness such as hardening behavior due to geometric nonlinearities or opponent magnets. Another test case is a cantilevered beam called RubBeR, which is strongly damped by dry friction [8]. This test rig is challenging due to the large increase in damping and frequency shift as well as a significant local mode shape change. The friction is caused by relative motion between the beam and fixed steel plates. The preload at the contact interface is set such that both full-stick and macro-slip is observed in the excitation range of the used shaker. The first bending mode's resonance frequency with full-stick contact is about 111.3 Hz, identified at



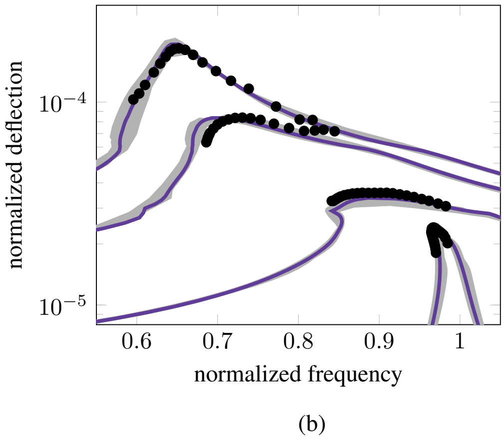


Figure 1: (a) Amplitude-dependent resonance frequency and modal damping ratio and (b) predicted (solid lines) and measured (dots) frequency responses of a friction-damped beam.

low excitation levels. The amplitude-dependent resonance frequencies and modal damping ratios are shown in Fig. 1a. The resonance frequency is normalized with the full-stick frequency, and the deflection amplitude is the amplitude at the beam's tip normalized with the beam's length. The well-known characteristics for friction-damped systems are observed: For low amplitudes, the resonance frequency is constant, but decreases with increasing vibration amplitude, here for normalized deflections above $2 \cdot 10^{-5}$. At the same time, the modal damping ratio increases drastically and decreases again for large amplitudes. For this specimen, the decrease in frequency is about 36 % and the modal damping ratio increases from about 0.1 % to about 15 %. By measuring backbones several times (not modifying the setup), the deviation of the modal properties due to the variance inherent to the test rig is assessed. The measurements are very well repeatable, leading to only a small spread of the extracted modal properties (gray-shaded areas in Fig. 1a with the average plotted with solid lines).

Assuming that the vibration energy is confined to a single nonlinear mode, the system behaves like a single degree-of-freedom oscillator. To assess the usefulness of the extracted properties, the oscillator is set up with the identified amplitude-dependent modal properties [6] and harmonically forced at four different excitation levels. The predicted frequency responses are plotted with solid lines in Fig. 1b and the predictions' spread based on the nonlinear modal properties' spread is indicated with the gray-shaded area. The reference frequency responses (black dots) were obtained with controlled stepped sine measurements. The deviation between predictions and reference measurements is small in comparison with the repeatability band. The very good agreement indicates that the nonlinear-modal oscillator with the properties shown in Fig. 1a is capable of describing frequency responses close to an isolated resonance.

Conclusions

In the absence of strong modal interactions, the proposed nonlinear modal testing method allows for extracting amplitude-dependent modal properties, i.e. resonance frequency, modal damping ratio and deflection shape, for systems with nonlinear dissipation. For all tested specimens, the single nonlinear-mode oscillator based on the identified modal properties describes well the steady-state dynamics around an isolated resonance. Therefore, a simplified single-point excitation mechanism with phase control of the fundamental harmonic or feeding back a scaled velocity signal is sufficient to extract accurate modal properties of structures with nonlinear dissipation. The concept of local phase resonance is, however, applicable only to structures with a diagonal dominant mass matrix (e.g. slender structures). Using single-point excitation for velocity feedback can cause gyroscopic forces that deteriorate the mode isolation quality.

References

[1] Noël J. P., Kerschen G. (2017) Nonlinear system identification in structural dynamics: 10 more years of progress. *Mechanical Systems and Signal Processing* **83**:2-35.

[2] Peter S., Leine R.I. (2017) Excitation Power Quantities in Phase Resonance Testing of Nonlinear Systems with Phase-Locked-Loop Excitation. *Mechanical Systems and Signal Processing* **96**:139-158.

[3] Denis V., Jossic M., Giraud-Audine C., Chomette B., Renault A., Thomas O. (2018) Identification of nonlinear modes using phase-locked-loop experimental continuation and normal form. *Mechanical Systems and Signal Processing* **106**:430-452.

[4] Renson L., Gonzalez-Buelga A., Barton D. A., Neild S. A. (2016) Robust identification of backbone curves using control-based continuation. *Journal of Sound and Vibration* **367**:145-158.

[5] Krack M. (2015) Nonlinear modal analysis of nonconservative systems: Extension of the periodic motion concept. *Computers and Structures* **154**:59–71.

[6] Scheel M., Peter S., Leine R.I., Krack M. (2018) A Phase Resonance Approach for Modal Testing of Structures with Nonlinear Dissipation. *Journal of Sound and Vibration* **435**:56–73.

[7] Scheel, M. (2022) Nonlinear modal testing of damped structures: Velocity feedback vs. phase resonance. *Mechanical Systems and Signal Processing* **165**:108305

[8] Scheel M., Weigele T., Krack M. (2020) Challenging an experimental nonlinear modal analysis method with a new strongly friction-damped structure. *Journal of Sound and Vibration* **485**:115580