Stochastic Response of Hopf Adaptive Frequency Oscillator

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Summary. Although not studied extensively, adaptive frequency oscillators (AFOs) could have many useful applications. AFOs possess the capability of synchronizing their oscillating frequency with their input frequency. Here, the noise-influenced dynamics of the Hopf Adaptive Frequency Oscillator (HAFO) are analyzed in a probabilistic manner. By adding a stochastic forcing term to the ordinary differential equations (ODEs), the resulting stochastic differential equations (SDEs) are integrated using the Euler-Maruyama (EM) method to obtain direct numerical solutions and the probabilistic dynamics of the oscillator. Additionally, a hardware circuit realization of the HAFO is fabricated, and the experimental results and the simulation results are compared. Efforts are made to quantify the working capability of the oscillator, which is limited by nonideal electrical components. The influence of noise on the HAFO circuit will also be investigated and compared with the results obtained through the Euler-Maruyama simulations.

Hopf Adaptive Frequency Oscillator

The Hopf Adaptive Frequency Oscillator is capable of synchronizing its oscillating frequency to an oscillatory input signal. The HAFO with the capability of learning the frequency of any rhythmic inputs are widely used for robotic locomotion control, by using the HAFOs as central pattern generators to tune the walking patterns in a cooperative way [1, 2]. The HAFO is an augmented form of the Hopf oscillator [3], which has an additional state related to the frequency:

$$\begin{aligned} \frac{dx}{dt} &= (\mu - r^2)x - \omega y + kF(t) \\ \frac{dy}{dt} &= (\mu - r^2)y + \omega x \\ \frac{d\omega}{dt} &= -sgn(y)kF(t) \end{aligned} \tag{1}$$

where $r = \sqrt{x^2 + y^2}$, k is the amplitude of the deterministic forcing function, μ is a constant related to the limit cycle amplitude, and $F(t)$ is a sinusoidal forcing function. The first two equations are the typical version of the Hopf oscillator, while the $\frac{d\omega}{dt}$ equation allows frequency adaptation. The learning process is embedded into the dynamical system, and there are not any pre- or post-processing procedures needed to accomplish the frequency synchronization. This behavior may be observed in Fig. 1.

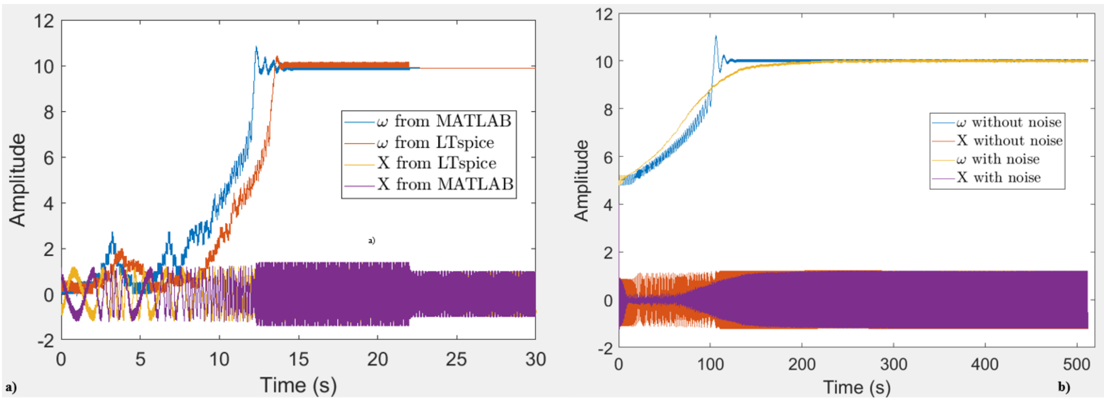

Figure 1: a) MATLAB simulation and LTspice simulation of eqs. 1. The sinusoidal forcing causes the HAFO to adapt. After the forcing is set to zero at t = 22 s, the HAFO "remembers" the input frequency. Additionally, the x output from both simulations overlaps with each other when the frequency adaptation is accomplished. b) MATLAB simulation of eqs. 2 by the Euler-Maruyama method. The noise amplitude is 10% of the amplitude of the sinusoidal forcing function. With the addition of noise, the frequency adaptation takes a longer time, but there is no overshoot. The x output from both cases overlaps with each other after the transient response.

Influence of Noise

Previously, the stochastic response of the HAFO was approximated by using a Fokker-Plank formulation [4]. As noise can change the dynamic stability of nonlinear systems [5, 6], it is important to further explore the effects of noise on the HAFO. To consider the effects of noise on the HAFO, the sinusoidal function, $F(t)$, is replaced with $\hat{F}(t) + \frac{\sigma}{k}\dot{W}(t)$. Here, $\hat{F}(t)$ is a sinusoidal function, $\dot{W}(t)$ is white Gaussian noise, and σ is the amplitude of the noise. Making this replacement, the stochastic differential equations are:

$$\begin{aligned} \frac{dx}{dt} &= (\mu - r^2)x - \omega y + k\hat{F}(t) + \sigma\dot{W}(t) \\ \frac{dy}{dt} &= (\mu - r^2)y + \omega x \\ \frac{d\omega}{dt} &= -sgn(y)k\hat{F}(t) - sgn(y)\sigma\dot{W}(t) \end{aligned} \tag{2}$$

This set of SDEs can then be simulated using the Euler-Maruyama method, depicted in Fig. 1.

Circuit Realization

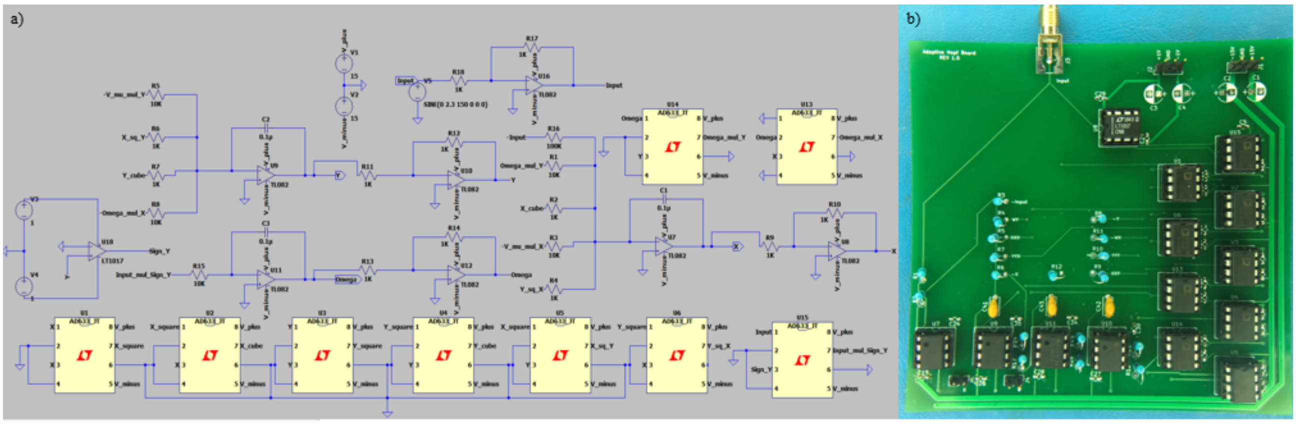

Figure 2: a) The circuit diagram in LTspice. b) A printed circuit board (PCB) of the HAFO.

The circuit design (Fig. 2) was inspired from [7], which presents an electronic implementation of the Lorenz chaotic oscillator for radar applications. To make the hardware circuit easier to implement, the original set of equations is modified, as in [8]:

$$\begin{aligned}
\frac{dx}{dt} &= (\mu - r^2)x - \omega y + kF(t) \\
\frac{dy}{dt} &= (\mu - r^2)y + \omega x \\
\frac{d\omega}{dt} &= -\frac{y}{\sqrt{x^2+y^2}}kF(t)
\end{aligned} \tag{3}$$

The modification of eqs. 1 does not change the frequency adaptation property. However, it does affect the length of the transient response and the error (the difference between the steady-state frequency of the oscillator and the frequency of the input signal). In Fig. 3, a comparison between the experimental circuit and the LTspice simulation is shown. Filtering was performed on the experimental data in MATLAB. Nonideal electronic components cause discrepancies between the experimental circuit and the simulated circuit. The voltages reported in the figure must be converted to find the frequency in Hertz.

Conclusions

AFOs could prove to have useful properties for mechatronics. To gain better understanding of their dynamics, numerical simulations of the deterministic and stochastic system are pursued, while a PCB implementation provides experimental insight. In Fig. 3, the experimental frequency was recorded with an oscilloscope, and LTspice was used to simulate the HAFO circuit. The variation between the experimental results and the numerical results is lower than 5%. The modeling of the HAFO on the hardware circuit achieved by the PCB is practical and inexpensive. Further work will be pursued to determine their efficacy as analog controllers.

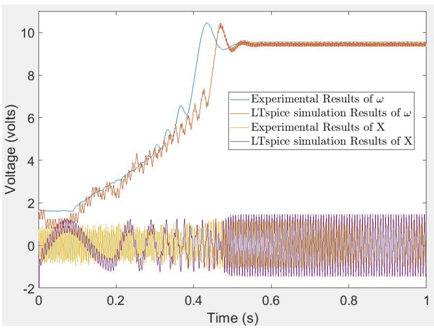

Figure 3: Comparison between the experiment and simulation.

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