Passive suppression of parametric excitation of cables using a nonlinear vibration absorber (NVA)

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Summary. The usage of non-linear vibration absorbers is a rather new but practical way to prevent a fatigue failure of a structure exposed to especially broad banded dynamic loading. In this work, a non-linear vibration absorber will suppress the response of a flexible rod under parametric excitation. Therefore, a mathematical model is derived using the extended Hamilton's principle. A reduced-order model (ROM) is obtained after the application of the Galerkin's method and is numerically integrated aiming at observing the Mathieu's instability and its suppression. This excitation scenario is common in offshore engineering where parametric excitation appears on risers of TLP tethers, slender structures linked to the seabed and the floating unit. Nonlinear vibration absorbers can be designed, built and installed with a relatively small effort compared to other methods and provide a positive effect on the sustainability of the rod [1]. The results show a suppression of the oscillation is possible to a certain degree, when the NVA reaches a sufficient mass ratio.

Introduction

Besides the academic interest, the studies of parametric excitation have a technological importance on off-shore and ocean engineering. Especially for the dynamic of risers and tendons of TLPs (Tension Leg Platforms), buoyant platforms held in place by a mooring system as can be seen in figure 1. Even though the considered structures are always under tension, the value varies with time due to the vertical movement of the platforms and the change of lifting force.

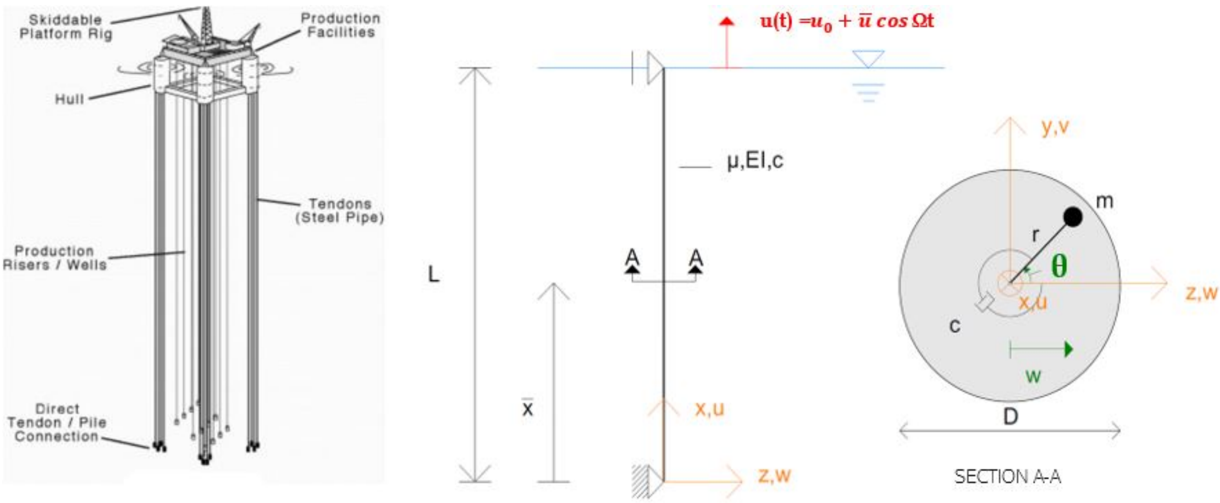

Figure 1: TLP [4] and system of flexible cable with NVA at x $= \bar{x}$

Parametric excitation

In contrast to external excitation, parametric excitation leads to *homogenous* differential equations with a parameter depending only on time on the left side. Of significance is that even a small parametric excitation produces a large response when the frequency of excitation Ω resembles twice the natural frequency ω of the system (*principle parametric resonance*)[2]. Therefore, an efficient suppression device is to be designed.

Non-linear suppression

A non-linear vibration absorber (NVA) as can be seen in figure 1 has the advantage to be able to react to a broad band loading. Due to the absence of a linearizable natural frequency, the NVA uses internal resonances for a spacial transfer of energy and its dissipation. This process is known as energy pumping, or TET (Targeted Energy Transfer). Further information can be found in papers by Lee [3] and Vakakis [5].

Mathematical model

Consider a flexible cable with a diameter D, mass per unit length μ and the structural damping ratio c. A rotative NVA (mass m, radius r) is attached to the cable at the height $\bar{x}$ by a dashpot (c_Θ), as can be seen in figure 1. The kinetic energy T and the potential energy V with $\epsilon = u' - zw'' + \frac{1}{2}w'^2$ are regarded for the Lagrangian, as well as the virtual work for extended Hamilton's principle. Using Galerkin's method by considering one degree of freedom (1-DOF) and placing the NVA at midspan, the non-dimensional equations of motion can be written as seen in equation 1 for the system and equation 2 for the NVA.

$$(\frac{1}{2}+\hat{m})\ddot{A}_1+\zeta_s\dot{A}_1+K_3\dot{A}_1|\dot{A}_1|+[K_0+\bar{K}cos\eta\tau]A_1+K_2A_1^3=\hat{r}\hat{m}(\ddot{\Theta}sin\Theta+\dot{\Theta}^2cos\Theta) \quad (1)$$

$$\ddot{\Theta}-\frac{1}{\hat{r}}sin\Theta\ddot{A}_1+2\zeta_\Theta\dot{\Theta}=0 \quad (2)$$

Results and discussion

After a numerical Runge-Kutta based integration with MATLAB, the behaviour of structure and NVA can be observed. In figure 2 the time histories can be divided in three main sections, where A and B show the typical behaviour of energy pumping, while C describes a more or less steady oscillation. Figure 2 also shows that depending on given parameters a NVA might also lead to an increase of the main amplitude. To give an overview over the choice of parameters $\hat{m}$ and $\hat{r}$,

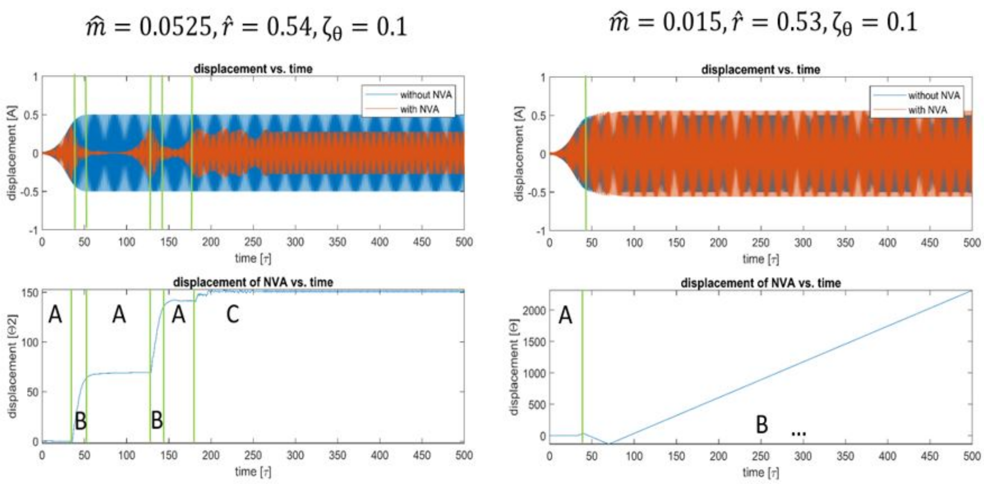

Figure 2: Time history and NVA sections with $\hat{m}=\frac{\text{mass NVA}}{\text{mass cable}}$, $\hat{r}=\frac{r}{D}$

the ratio of standard deviations with and without NVA can be seen in figure 3, where red areas show an increase and blue areas a significant reduction of the main structures vibration.

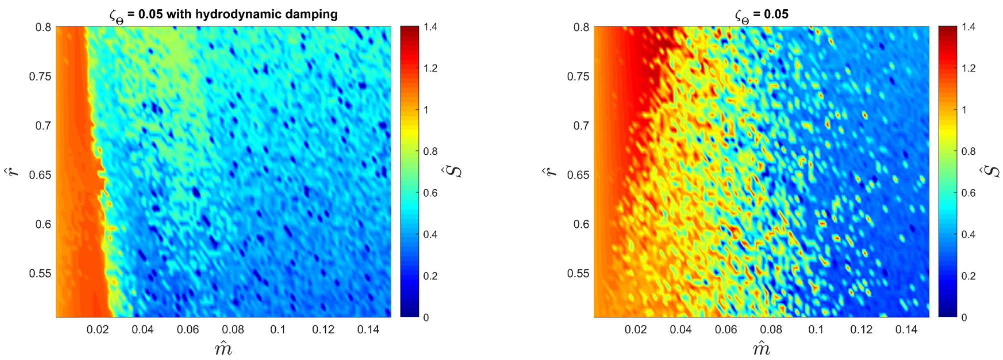

Figure 3: Maps

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