A new explicit CD-Lagrange scheme with redistributed mass for structural dynamics with impacts

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Summary. This work aims to apply mass redistribution method to explicit schemes in non-smooth contact dynamics, especially the CD-Lagrange scheme. New properties are being investigated to better address the issues of impact for structural transient dynamics. Mass redistribution method improves energy balance and stability for CD-Lagrange scheme. Results for a simple contact problem are presented.

Non-smooth Contact Dynamics and mass redistribution method

In structural transient dynamics, problems with impact belong to the family of Non-Smooth Contact Dynamics (NSCD) problems. An impact causes a discontinuity in velocity, and therefore a non-defined acceleration. Integrating in time requires thus dedicated schemes. A first family, event-driven schemes, adjust the time-step to set impact times at discrete times. They fail in case of numerous and close impacts leading to a high computation time. A second family, time-stepping schemes, can deal with multiple impacts between two discrete times keeping time-step constant. Time-stepping schemes are thus more robust for practical finite elements problems.
The first NSCD time-stepping scheme is Moreau-Jean's scheme [1, 2]. It introduces a new contact formulation written in terms of velocity and impulsion. It allows stability at impact and a better energy balance, and is equivalent to classical Signorini's condition. But writing contact in terms of velocity does not solve exactly position. A residual penetration remains during contact. Chen, Acary et al. [5] extend Moreau's velocity formulation to Newmark's family. All these schemes are implicit.
A first family of NSCD explicit schemes is Paoli-Schatzman's one [3]. They use a contact formulation in terms of position, and central difference method for time-integration. For velocity/impulse contact formulation, the only explicit scheme is CD-Lagrange [4]. It uses the central difference method too for time-integration, but preserves properties of Moreau-Jean's family.
A active research field in NSCD is currently the mass redistribution method. Following the work of Khenous, Laborde and Renard [6], new spatial discretizations appear with massless node for contact. This improves the energy balance, and velocity stability for contact nodes after release. Dabaghi [7] applies this technique to Paoli-Schatzman's schemes leading to similar improvements.
In the present work redistribution mass method is applied to CD-Lagrange scheme. The purpose is to exhibit the interest of mass redistribution method to velocity/impulse contact formulation and a explicit scheme. This new scheme would be well suited to non-linear impact dynamics involved in tire simulations.

CD-Lagrange scheme with redistributed mass : a 1D example

Figure 1: Redistributed mass on a discrete bar

Here a one-dimensional problem is presented to illustrate the main features of mass redistribution on a CD-Lagrange scheme. The impacting bar problem is chosen. It is a common benchmark case for NSCD integration schemes [4, 5, 7, 8]. This case is a discrete bar impacting on a rigid barrier, with a linear elastic material law.
Here the skin is defined as being the last element of the bar. Modifying mass consists of removing mass of contact node and distributing it uniformly on other nodes. It results in two problems : one for the bulk, and one for the skin. The bulk problem is dynamic with kinetic and internal stresses. The skin problem is similar to a static one. The contact node having no mass, no kinetical term is involved in equation of dynamic.
This leads to a difficulty for schemes with velocity as main unknown (like CD-Lagrange): the contact node velocity can not be determined by the equation of dynamic. The solution is to set the normal velocity depending on state of contact. It is fixed to zero for an active contact, and to the normal velocity of preceding mass node otherwise. This means that the skin deforms only when the contact is active. After release, a deformed state can thus persist. In such a case the skin force acting on bulk domain is set to zero. As a consequence the contact problem no longer contains kinetic terms, but only forces based on stresses in skin.

Figures 2 and 3 show respectively the contact node position and velocity over time for analytical, classical CD-Lagrange and redistributed mass CD-Lagrange solutions.
The analytical solution of the continuous problem is known. The bar gets closer to the barrier with constant speed and impacts it (around $t = 0,5 \times 10^{-4}s$). The contact persists until the compression wave reaches the free end and returns.

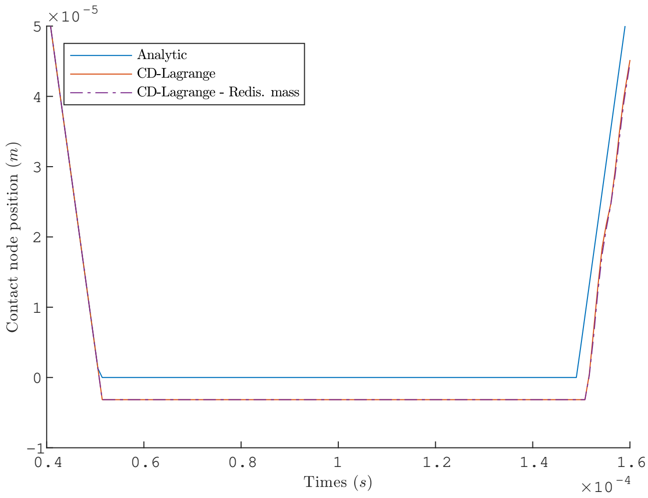

Figure 2: Contact node position

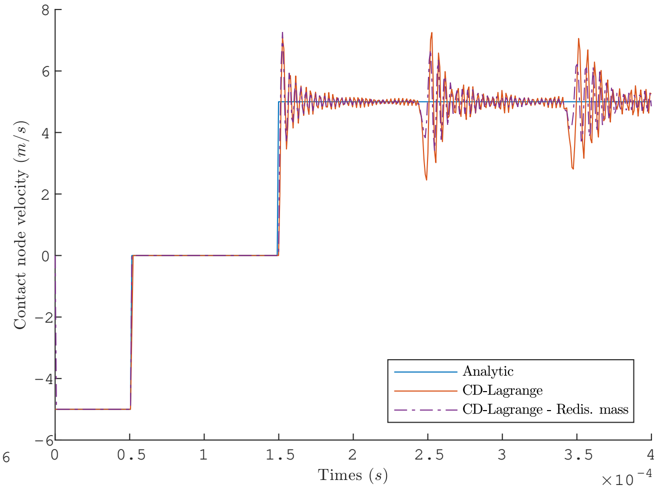

Figure 3: Contact node velocity

Then the bar releases contact (around $t = 1.5 \times 10^{-4} s$) and moves away with the same constant speed as before impact.

Time-integration schemes compute discrete solutions with commons features : a residual penetration remains after impact, because of the velocity contact condition. This residual penetration is the same for both CD-Lagrange schemes. Redistributing mass has no influence on discrete position. For velocity, if the discrete solutions stay close from analytic one before the contact release, spurious oscillations appear after. For redistributed mass CD-Lagrange, they vanish slightly faster.

Discrete energy over time is depicted on figure 4. This energy contains only conservative terms: kinetic, internal and complementary energy. At impact time the loss of energy for CD-Lagrange is the kinetic energy of contact node, as its velocity is set to zero. For CD-Lagrange with redistributed mass, no energy is lost as the node is massless. But at contact release, some energy is injected.

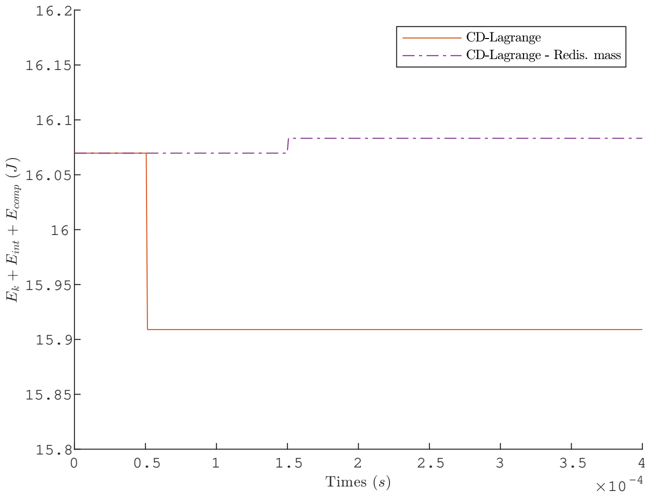

Figure 4: Energy : conservative terms

Conclusion and future work

Embedding mass redistribution in CD-Lagrange scheme allows to improve three features: a new contact problem, simpler without kinetic terms; a conservative impact; a weakening of spurious oscillations slightly improved.

The next step of this work is to extend it to three-dimensional problems. A major issue is dealing with incompatible meshes, that leads to a higher complexity for the impact problem.

References

[1] Moreau J. J. (1999) Numerical aspects of the sweeping process. *Comput. Methods Appl. Mech. Eng.* vol. 177, no. 3-4, pp. 329-349

[2] Jean M.(1999) The non-smooth contact dynamics method. *Comput. Methods Appl. Mech. Eng.* vol. 177, no. 34, pp. 235-257

[3] Paoli L., Schatzman M. (2002) A Numerical Scheme for Impact Problems I: The One-dimensional Case. *SIAM J. Numer. Anal.* vol. 40, no. 2, pp. 702-733

[4] Fekak F. E. , Brun M., Gravouil A., Depale B. (2017) A new heterogeneous asynchronous explicit-implicit time integrator for nonsmooth dynamics. *Comput. Mech.* vol. 60, no. 1, pp. 1-21

[5] Chen Q., Acary V., Virlez G., Brüls O. (2013) A nonsmooth generalized-α scheme for flexible multibody systems with unilateral constraints. *Int. J. Numer. Methods Eng.* vol. 96, no. 8, pp. 487–511

[6] Khenous H. B., Laborde P., Renard Y. (2008) Mass redistribution method for finite element contact problems in elastodynamics, *Eur. J. Mech. A/Solids* vol. 27, no. 5, pp. 918-932

[7] Dabaghi F., Petrov A., Pousin J., Renard Y. (2016) A robust finite element redistribution approach for elastodynamic contact problems *Appl. Numer. Math.* vol. 103, pp. 48-71

[8] Di Stasio J., Dureisseix D., Gravouil A., Georges G., Homolle T. (2019) Benchmark cases for robust explicit time integrators in non-smooth transient dynamics, *Adv. Model. Simul. Eng. Sci.* vol. 6, no. 2

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