Coupled Thermoelastic Large Amplitude Vibrations of Bi-Material Beams

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Summary. The goal of this work is to develop a theoretical and numerical models of the coupled and uncoupled vibration of bi-material beams subjected the combined action of mechanical and thermal loads. The geometrically nonlinear version of the Timoshenko beam theory is used to describe the theoretical model of the problem. Starting from the geometrical, constitutive and equilibrium equations of each layer the governing equations of the bi-material beam are derived. The beam is subjected to heat flux and periodic mechanical loading. The influence of the elevated temperature or the heat propagation along the beam length and thickness on the response of the beam was studied.

Introduction

Among the most popular composite structures the bi-material structures and especially the bi-material beams are frequently used in different braches in industry. The growing interest to mechanical behavior of the bi-material beam can be connected with different MEMS (see, for example https://istegim2019.sciencesconf.org/285421). The thermoelastic behaviour of such beams is a subject of interest because of their applicability as well as because of the complex behaviour of the structures due to the different elastic and thermal properties of the layers.

Generally, the most of the studies of the dynamic response of the thermally loaded beam consider that the structure gets elevated temperature instantly, and the heat propagation is not included in the model.

The goal of the present work is to derive the equations of the geometrically non-linear vibration of a bi-material beam with non-symmetric layers according to the Timoshenko beam theory. Based on these equations it is aimed to study the coupled and uncoupled geometrically nonlinear vibrations of the beam as well as the vibration of the beam at constant elevated temperature. The influence of the coupled terms in the governing equations and the new terms counting the non-uniform layers are specially studied for different material properties and different thermal loadings. The speed of the propagation of the temperature along the different layers is also analyzed. The reduced model created on the base of Galerkin approach allows analysis of the beam response in the frequency and time domains and an estimation of the stability of the solution, as well. Three dimensional finite element model of the bi-material beam is created in order to verify the results.

Theoretical model

A beam with length l, width b and thickness h is considered. The beam consists of two layers made of different materials (Material 1 and Material 2) with thickness h_1 and h_2 ($h=h_1+h_2$). The geometrical scheme of the beam is shown in Fig. 1.

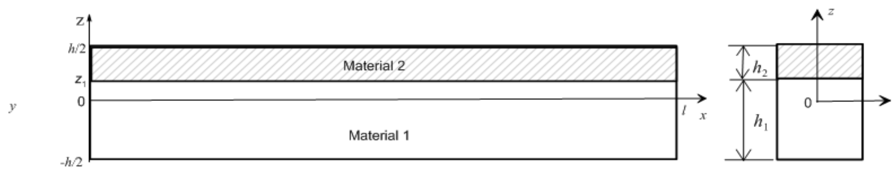

Fig. 1 The geometrical scheme of the beam model

The equations describing the coupled problem of the temperature propagation and the beam vibrations as a result of the action of a heat flow and of mechanical load with intensity $p(x,t)$ are:

$$\frac{c_p^{(i)}}{\lambda_T^{(i)}}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}-\frac{\alpha_T^{(i)}E^{(i)}T_0}{\lambda_T^{(i)}}\frac{\partial \varepsilon^{(i)}}{\partial t},\quad i=1,2$$

$$\frac{\partial N}{\partial x}-c_1^{(i)}\frac{\partial u}{\partial t}-\rho^{(i)}bh^{(i)}\frac{\partial^2 u}{\partial t^2}=0\,,\ -\frac{\partial M}{\partial x}+Q-c_2^{(i)}\frac{\partial \psi}{\partial t}-\rho^{(i)}I^{(i)}\frac{\partial^2 \psi}{\partial t^2}=0,$$

$$\frac{\partial Q}{\partial x}+N\left(\frac{\partial^2 w}{\partial x^2}\right)+\frac{\partial N}{\partial x}\frac{\partial w}{\partial x}-c_1^{(i)}\frac{\partial w}{\partial x}-\rho F\frac{\partial^2 w}{\partial t^2}=-p(x,t)$$

where $I^{(i)}$ is the inertia moment of i^{th} layer, $\rho^{(i)}$ is the density of the i^{th} material $T(x,z,t)$ is current temperature, T_0 is the initial constant temperature, $\lambda_T^{(i)}$ is the thermal conductivity of i^{th} material and $c_p^{(i)}$ is the heat capacity per unit volume,

$\alpha_T^{(i)}$ is the coefficient of the thermal expansion, $E^{(i)}$ is the Young's modulus of the i^{th} layer, $c_1^{(i)}$ and $c_2^{(i)}$ are damping coefficients, $w(x,t)$ is the transverse displacement, $\psi(x,t)$ is the rotation angle and

$$\varepsilon^{(i)} = -z\frac{\partial \psi}{\partial x} + \alpha_T^{(i)}\left(T - T_0\right) + \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$

After integrations along each layer thickness the generalized stresses *N*, *M* and *Q* are expressed by the displacements and then the governing equations of the bi-material beam are derived.
In comparison with the classical Timoshenko beam equations, additional nonlinear terms appear in the equations describing the longitudinal displacements, transverse displacements and the angular rotations.

Numerical approach

The equation for the heat propagation is discretized with respect to the space variables (*x* and *z*) by the finite difference method. The partial differential equations for the beam vibration are transformed to a system of coupled nonlinear ordinary differential equations by the Galerkin approach using the expansion of the generalized displacement vector in a series of the product of the normal modes of the linear Timoshenko beam and time dependent coefficients. The algorithm for the solution of the problem is based on the successive solution of the equations for the mechanical vibrations of the beam and for the heat transfer. The algorithm is similar to the one described in [1].

Numerical examples

Two cases of the problem have been studied: (i) beam at elevated temperature and (ii) beam subjected to a heat impact. The influence of the elevated temperature applied together with periodic loading on the beam is studied in time and frequency domain.
In the case of the heat impact the propagation of the temperature along the beam layers and specially at the interface layer is studied. It is shown that in the case of short and intensive heat pulses the beam can buckle (Fig. 2). The influence of the nonlinear terms appeared due to the different properties of the materials is estimated.
In some cases, the coupled terms may influence the temperature propagation as can be seen in Fig. 3.

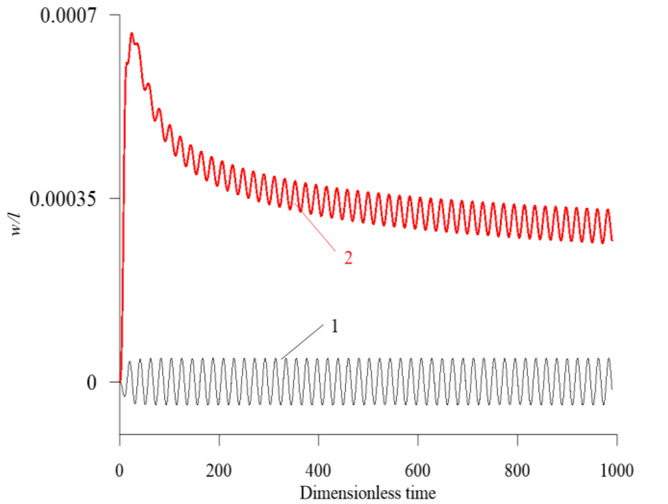

Fig. 2. Time history of the response of the beam subjected to thermal and harmonic loading with 1- no heating; 2-heat impact with duration $\bar{t}_0$ =30 (dimensionless time) and q_0=9000 W/m^2.

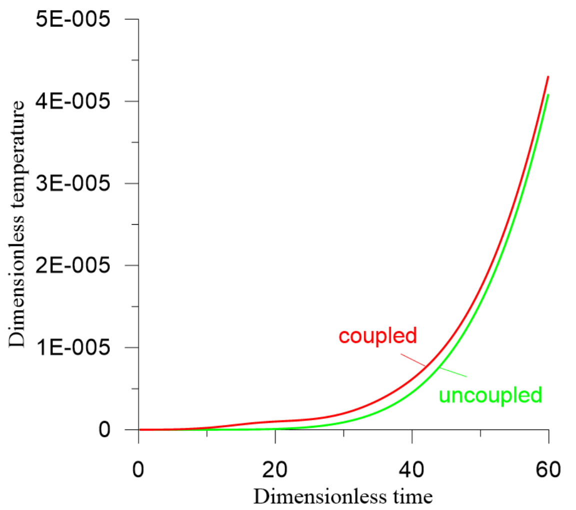

Fig. 3. Variation of the temperature at the beam center at 6th interface layer of the beam cross-section.

Conclusions

A theoretical model of the dynamic behaviour of Timoshenko beam is developed. The model includes a full coupling of the mechanical and thermal fields i.e. the thermal field influences the beam motion and the beam motion influence the heat propagation. The dynamic behavior of the beam is studied in the case of coupled, uncoupled vibration, as well as the case of elevated constant temperature. 3D finite element model of the problem and a reduced model are used and the results obtained by both models are compared.

References

[1] Manoach E, Ribeiro, P. (2004) . Coupled, thermoelastic, large amplitude vibrations of Timoshenko beams. Int. J. Mechanical Sciences, 46, 1589-1606 .