Passive suppression of axial-flow-induced vibrations of a cantilevered pipe discharging fluid using non-linear vibration absorbers

Vitor Schwenck Franco Maciel[†], Mojtaba Kheiri[‡] and Guilherme Rosa Franzini[†]
[†] *Offshore Mechanics Laboratory, Escola Politécnica, University of São Paulo, Brazil*
[‡] *Fluid-Structure Interactions & Aeroelasticity Laboratory, Concordia University, Canada*

Summary. The present work addresses passive suppression of vibrations induced by internal axial flow in a cantilevered flexible pipe discharging fluid. The suppressor utilized is a rotative non-linear vibration absorber (NVA) composed of a mass connected to the extremity of a rigid bar hinged to the main structure by means of a dashpot. Numerical results from the mathematical model show that the NVA is able to bound the pipe structural response even in the supercritical flow regime.

Introduction

Today, the problem of a cantilevered pipe conveying fluid is considered as a classical problem in the study of the dynamics and stability of structures because of its simplicity and potential for displaying a wide range of complex dynamics. This problem belongs to a broader class of open dynamical systems with axial momentum transportation, and several studies, such as [1] and [2], have been made on the extension of Euler-Lagrange's equations and Hamilton's principle for such systems. Pipes conveying fluid, in their myriad of applications, such as heat exchangers and risers are susceptible to flow-induced instabilities and vibrations which in turn can lead to fatigue failure, excessive noise and leaks.

Non-linear vibration absorbers (NVAs) have been studied in the last decades as alternatives to linear suppressors, such as the tuned mass damper (TMD), for passive suppression of oscillations. Even though the literature on the use of NVAs for suppressing axial-flow-induced vibrations is recent, studies have shown that they are capable of adapting and effectively operating even if the load is broadbanded due to an energy transferring concept called Targeted Energy Transfer (TET), as documented in [3] and [4].

This paper focuses on the use of a particular class of suppressors, called rotative NVA, with the objective of suppressing axial-flow-induced vibrations of cantilevered pipes discharging fluid. Such a suppressor is composed of a point mass connected to the extremity of a rigid bar which, in turn, is hinged to the pipe via a dashpot. In [3], the authors discuss passive suppression of internal-flow-induced oscillations of a pinned-pinned pipe using a different type of NVA. To the best of the authors' knowledge, the use of a rotative NVA for suppressing axial-flow-induced vibrations of pipes conveying fluid has not been previously addressed and is the main novelty of the present work.

Mathematical model

Consider a flexible pipe discharging fluid as shown in Fig. 1a. The pipe has the length L, the diameter D, the bending stiffness EI and the mass per unit length m. The fluid has the mass per unit length M and flows at a constant velocity U. At a point $\bar{x}$ along the pipe length, the NVA is placed and constrained to rotate in the (y, z)-plane. The device has the mass m_n, the radius r and the damping constant c. The pipe transverse displacement is represented by $w(x, t)$ while the angular displacement of the NVA is $\theta(t)$. It is assumed that the pipe is inextensible and that the flow is incompressible and has a uniform profile in the pipe (i.e. the plug flow model). By using the extended Hamilton's principle found in [5], the dimensional equations of motion can be found and are made dimensionless using the following quantities:

$$\tau = \left(\frac{EI}{m+M}\right)^{1/2}\frac{t}{L^2},\ \xi = \frac{x}{L} = \frac{s}{L},\ \eta = \frac{z}{L} = \frac{w}{L},\ \gamma = \frac{(m+M)gL^3}{EI},\ \hat{\delta}(\xi-\bar{\xi}) = L\delta(x-\bar{x}),$$
$$\hat{c} = \frac{cL^2}{m_n\left(\frac{EI}{m+M}\right)^{1/2}r^2},\ u = \left(\frac{M}{EI}\right)^{1/2}UL,\ \hat{m}_n = \frac{m_n}{(m+M)L},\ \beta = \frac{M}{(m+M)},\ \hat{L} = \frac{L}{D},\ \hat{r} = \frac{r}{D}, \qquad (1)$$

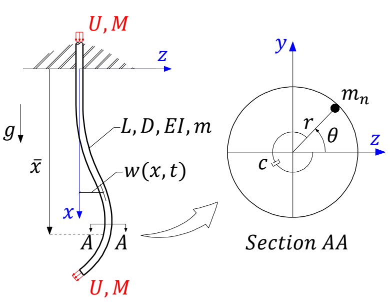
where s is the curvilinear coordinate along the pipe length, and δ is the Dirac Delta function. The resulting dimensionless partial differential equation is then discretized using Galerkin's method, with the adoption of the first five mode shapes of a cantilevered Euler-Bernoulli beam, i.e., $\eta(\xi,\tau) \cong \sum_{n=1}^{5} A_n(\tau)\psi_n(\xi)$, where ψ_n are the mode shapes and A_n are the corresponding generalized coordinates. Using the notation $(\ )' = \partial/\partial\xi$ and $\dot{(\ )} = \partial/\partial\tau$, the final system of six coupled second-order ordinary differential equations is obtained in the form of equations (2) and (3), for $k = 1, ..., 5$.

Results

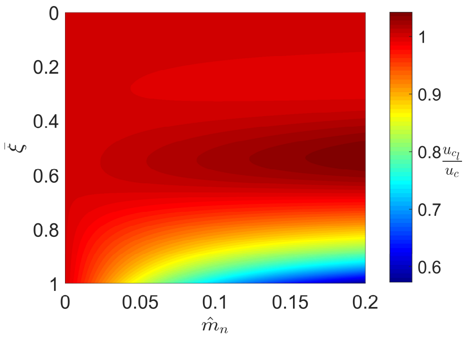
Consider three different systems: system 1 is the unaltered (or plain) pipe, system 2 is composed of the pipe and a lumped mass rigidly attached to the pipe at different spanwise locations and, finally, system 3 is the one with the NVA. System 2 allows for investigations into the effect of the placement of the stationary lumped mass on the critical flow velocity – "static" effect – via Lyapunov's Indirect Method. On the other hand, system 3 allows for investigations into the capability of the device to mitigate vibrations of the main structure due to its motion with respect to the pipe, which locally dissipates energy in the associated dashpot (TET mechanism). Here, passive suppression is achieved by a "dynamical" effect.

$$\left(\sum_{n=1}^{5}\int_{0}^{1}\psi_k\psi_n d\xi+\hat{m}_n\sum_{n=1}^{5}\int_{0}^{1}\hat{\delta}(\xi-\bar{\xi})\psi_k\psi_n d\xi\right)\ddot{A}_n+\left(2\beta^{\frac{1}{2}}u\sum_{n=1}^{5}\int_{0}^{1}\psi_k\psi_n' d\xi\right)\dot{A}_n+$$
$$+\left(\sum_{n=1}^{5}\int_{0}^{1}\psi_k\psi_n'''' d\xi+u^2\sum_{n=1}^{5}\int_{0}^{1}\psi_k\psi_n'' d\xi-\gamma\sum_{n=1}^{5}\int_{0}^{1}(1-\xi)\psi_k\psi_n'' d\xi+\gamma\sum_{n=1}^{5}\int_{0}^{1}\psi_k\psi_n' d\xi\right)A_n-$$
$$-\frac{\hat{r}}{\hat{L}}\hat{m}_n(\ddot{\theta}\sin\theta+\dot{\theta}^2\cos\theta)\int_{0}^{1}\hat{\delta}(\xi-\bar{\xi})\psi_k d\xi=0, \tag{2}$$
$$\ddot{\theta}-\frac{\hat{L}}{\hat{r}}\sin\theta\sum_{n=1}^{5}\ddot{A}_n\psi_n(\bar{\xi})+\hat{c}\dot{\theta}=0. \tag{3}$$

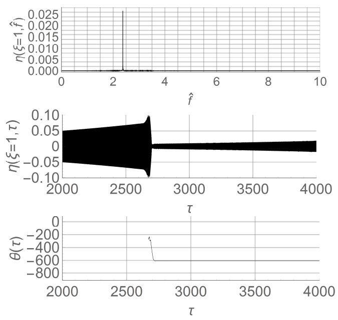
Throughout this extended abstract, we assume $\beta = 0.20$ and $\gamma = 10$. An example of the analysis on the "static" effect of the device is given in Fig. 1b, which shows the variation of u_{cl}/u_c (u_c and u_{cl} being the critical flow velocities for systems 1 and 2, respectively) as a function of the lumped mass value and its location. As seen, counter-intuitively, the lumped mass has a destabilizing effect with the exception of the region approximately defined by $\bar{\xi} \in [0.35, 0.65]$ and $\hat{m}_n \in [0.025, 0.2]$, in which the critical flow velocity ratio u_{cl}/u_c increases as $\hat{m}_n$ is increased. For system 3, the "dynamical" effect is then evaluated through a numerical integration of the equations of motion at $u = u_{cl}$, which leads to unbounded responses of system 2. Considering $A_1(0) = 0.1$ and $\theta(0) = 0.1$ as the non-trivial initial conditions, Fig. 3c presents the time-history of the displacement at the free end of the pipe, i.e. $\eta(\xi = 1, \tau)$, the associated amplitude spectrum and the angular response of the NVA, $\theta(\tau)$, for the case in which the suppressor is placed at $\bar{\xi} = 0.5$ and is characterized by $\hat{m}_n = 0.01$, $\hat{c} = 0.2$ and $\hat{r} = 0.6$. Both time series are shown within $\tau \in [2000, 4000]$ range. Note that the response is bounded, even though for a supercritical internal flow velocity for system 2.



a)



b)



c)

Figure 1: a) Schematic drawing of the system, b) (u_{cl}/u_c) as a function of $(\hat{m}_n; \bar{\xi})$ - System 2, c) Example of response - System 3.

From Fig. 3c, a strongly modulated response composed of two intermittent regimes can be observed. The first regime is characterized by a growth in the pipe response, while the device oscillates around the positions that are aligned with the pipe motion, i.e. where $\cos\theta = \pm 1$. Then, when the energy reaches a certain threshold, the NVA rotates with practically the same frequency as the oscillation frequency, that is $\hat{f} = 2.37$. Hence, we may conclude that the observed passive suppression is associated with a $1:1$ resonance.

Conclusions

A rotative NVA was utilized to successfully mitigate vibrations of a cantilevered flexible pipe discharging fluid. Both the "static" and "dynamical" effects were examined. While the former showed to have an important role in the stability of the system (i.e., the critical internal flow velocity), the latter was responsible for bounding the dynamical response due to energy dissipation. More numerical results along with a more comprehensive analysis of the static and dynamic effects will be presented in the full-length paper.

Acknowledgements

The authors acknowledge FAPESP for the grants 19/27855-2 and 2021/04434-1 and CNPq for the grants 305945/2020-3 and 161955/2021-5.

References

[1] Benjamin T.B., and Batchelor, G. K. (1962) Dynamics of a system of articulated pipes conveying fluid - I.Theory. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* **457-486**

[2] Casetta L. and Pesce C.P. (2013) The generalized Hamilton's principle for a non-material volume. *Acta Mechanica* **919–924**

[3] Yang T., Liu T., Tang Y., Hou S., and Lv X. (2018) Enhanced targeted energy transfer for adaptive vibration suppression of pipes conveying fluid. *Nonlinear Dynamics* **1937-1944**

[4] Lee Y.S. et al. (2008) Passive non-linear targeted energy transfer and its applications to vibration absorption: a review. *Journal of Multi-Body Dynamics*

[5] Païdoussis M.P. (2014) Fluid-Structure Interactions and Axial Flow. Volume 1, 2nd Edition, *Elsevier Science*