Nonlinear oscillations of a beam-like model of pipe with deformable cross-sections

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Summary. Nonlinear dynamics of a beam-like model of pipe are considered here. The mechanical model allows change in shape of the cross-sections of the pipe under bending, through a nonlinear and coupled response function for the hyperelastic materials, which is outcome of a homogenization procedure. The coupling between global bending and change in shape of the cross-sections is addressed, and large amplitude vibrations are analyzed under harmonic loading, including different internal resonance conditions.

Introduction

Pipes are widely used structures in civil and industrial applications, and the evaluation of their strength represents a key point for structural engineers to be addressed in the design process. Usually, their thin-walled nature introduces some peculiar features to be considered during structural analysis, like possible occurrence of local effects as well as change in the transversal shape. In this framework, the use of beam-like models for their analysis can, on the one hand, allow one to deal with (moderately) handy equilibrium or dynamic equations; on the other hand, they may not avoid to include specific aspects which are commonly discarded for slender beams, like cross-section change in shape and warping. For instance, it is well known that ovalization of the cross-section, taking places when pipes are bent over, may lead to instability phenomena, with consequent loss of carrying capacity of the whole structure. This phenomenon is explained by the occurrence of a limit point in the bending moment-curvature relationship, as a consequence of the softening nature of such a law, and is known as Brazier effect [1]. This specific effect is analyzed in [2] in case of anisotropic materials. In case of multi-layered structures, a one-dimensional beam-like, which is derived from a corresponding three-dimensional continuum, is proposed in [3], distinguishing the case of open and closed cross-sections and accounting for the Vlasov theory. Theories of multi-layered beams and cross-sectional models are addressed in [4], where use of the Variational Asymptotic Method is made to obtain the equations of motion. In [5], a homogeneous beam-like coarse model is adopted to describe the mechanics of thin-walled beams with possibility of cross-section distortion. In [6], static and free dynamic analysis of a homogeneous beam-like model is performed, after an identification procedure from a companion three-dimensional continuum. The same model is then extended in [7], where further parameters are introduced in order to take into account shear deformations in the multi-layered case, and in [8], where inertial contributions are identified in order to take into account dynamical effects.

In some cases, a soft core realized with structural foam may be introduced to improve the performance of the pipes under bending. For instance, in [9], the contribution of soft elastic cores is analyzed in thin-walled cylindrical structures, letting inspiration from nature where, e.g., plant stems or hedgehog spines have their mechanical efficiency increased by soft cores. In [10] the optimum design of thin-walled cylindrical shells with compliant cores, subjected to uniaxial compression and bending, is theoretically addressed and experimentally proved. In [11], an internally constrained beam model is proposed to deal with foam filled tubes, and equilibrium analysis is then performed in order to forecast and reproduce typical phenomena.

Here, the one-dimensional non-standard beam-like model presented in [7, 8] is used to address nonlinear oscillations under specific internal resonance conditions between global (bending) and local (ovalization) modes, also in presence of a resonant external load which expends work on cross-section flattening.

Equations of motion

A nonlinear beam-like model is used to analyze nonlinear dynamics of an elastic pipe. The beam model is shown in Fig. 1a, where the axis-line, spanned by the abscissa s running from 0 to l, and the generic cross-section are sketched. Besides the classical kinematic descriptors of planar Timoshenko beams, which are the longitudinal and transversal displacements of the axis line (u, v) and the cross-section bending rotation (ϑ), two further s- and time-functions are used, referred to as a_p and a_w and related to the cross-section change in shape, in its plane and out of its plane, respectively. The kinematic equations are posed and the dynamic equilibrium equations of the beam evaluated from the consequential use of the virtual power theorem. The constitutive relations are evaluated by means of a process of homogenization from a corresponding 3D model, shown in Fig. 1b, where the pipe is assumed as realized by longitudinal rods and transversal ribs. For the homogenization purposes, the change in shape of the cross-section is described as in the GBT theory [12], namely the kinematic variables a_p and a_w multiply assumed functions, as shown in Fig. 1c. The equations of motion are written

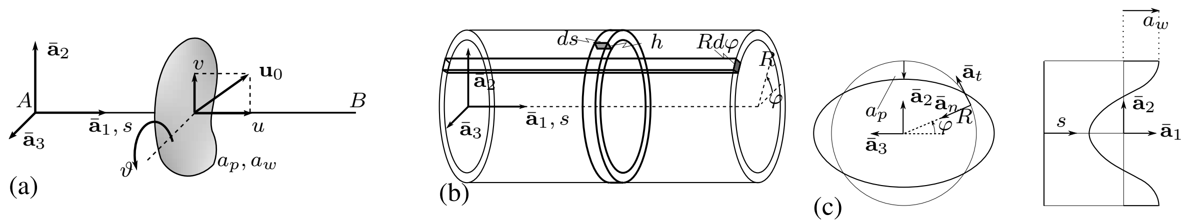

Figure 1: (a) The one-dimensional beam-like model; (b) the three-dimensional model with fibers and ribs; (c) the assumed functions and amplitudes for the change in shape of the cross-section.

below, where the nonlinear terms are related to elastic and inertial contributions:

$$\begin{aligned}&\left[c_1\left(v'-\vartheta-\left(\frac{\vartheta^2}{2}-\vartheta v'\right)\vartheta+\frac{1}{6}(\vartheta^3-3\vartheta^2 v')\right)-\frac{1}{2}c_1 a_w\vartheta'\right]'+p_v-m_1\ddot{v}\\&-\left\{\frac{c_1(v'-\vartheta)\vartheta^2}{2}-\vartheta\left[c_1(v'-\vartheta)\vartheta+\int_s^l\left(p_u-m_1\int_0^\xi\left(\frac{\vartheta^2}{2}-\vartheta v'\right)^{\bullet\bullet}d\zeta\right)d\xi\right]\right\}'=0\end{aligned}\tag{1}$$

$$\begin{aligned}&\left[c_2\vartheta'+c_3a_p\vartheta'+c_4a_p^2\vartheta'-\frac{1}{2}c_1a_w(v'-\vartheta)+\frac{1}{2}c_1a_w^2\vartheta'\right]'+c_1\left(v'-\vartheta-\left(\frac{\vartheta^2}{2}-\vartheta v'\right)\vartheta+\frac{1}{6}(\vartheta^3-3\vartheta^2v')\right)\\&-\frac{1}{2}c_1a_w\vartheta'+(\vartheta-v')\left(c_1(v'-\vartheta)\vartheta+\int_s^l\left(p_u-m_1\int_0^\xi\left(\frac{\vartheta^2}{2}-\vartheta v'\right)^{\bullet\bullet}d\zeta\right)d\xi\right)+c\\&-m_2\ddot{\vartheta}-m_3\dot{a}_p\dot{\vartheta}-m_3a_p\ddot{\vartheta}-m_4\dot{a}_pa_p\dot{\vartheta}-m_1a_w\dot{a}_w\dot{\vartheta}-m_5a_p^2\ddot{\vartheta}-m_6a_w^2\ddot{\vartheta}=0\end{aligned}\tag{2}$$

$$[c_7a_w+c_8a_w^3+\frac{1}{4}c_1a_p']'-c_5a_p-c_4a_p\vartheta'^2-c_6\vartheta'^2+q_p-m_4\ddot{a}_p+\frac{5}{8}m_6a_p\dot{\vartheta}^2-m_7\dot{\vartheta}^2=0\tag{3}$$

$$c_{12}a_w''-c_9a_w-c_7a_p'-c_{11}a_w^2a_p'-\frac{1}{2}c_1a_w\vartheta'^2-c_{10}a_w^3+\frac{1}{2}c_1(v'-\vartheta)\vartheta'+q_w-m_6\ddot{a}_w+m_6a_w\dot{\vartheta}^2=0\tag{4}$$

Boundary conditions are combined to Eqs. (1)-(4), prime indicates s-derivative and dot t-derivative, whereas p_v, c, q_p and q_w are external load components. It is worth noticing that the bending problem (variables v, ϑ) and the cross-section problem (variables a_p, a_w) are uncoupled in their linear part, while coupling occurs only due to nonlinear terms. Solutions are obtained both via perturbation methods and pure numerical approach, in case of internal resonance conditions among modes involving the bending and cross-section problems. Discussion on the effect of change in shape of the cross-sections during bending vibrations is provided, highlighting the resulting softening behavior related to reduction of bending stiffness of the pipe due to the cross-section flattening.

Conclusions

Nonlinear dynamics of a thin pipe is analyzed here. A homogeneous beam-like model is used, where specific kinematic functions describe the change in shape of the cross-section. Dynamic interaction between bending of the beam and change in shape of the cross-sections is addressed, leading to a softening behavior of the cantilever and related to the reduction of the bending stiffness as the cross-sections flatten.

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