Dynamics and Control of a Rotating Beam with Active Element

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Summary. Dynamics of a rotating flexible beam with an embedded active element is studied in the paper. The beam model is based on Bernoulli-Euler theory but it is extended taking into account beam's extensionality and large transversal deformations with nonlinear curvature. The macro fibre composite (MFC) embedded into the structure is applied to control or reduce externally excited vibrations. The model of MFC element is considered in a few variants of simplifications. The complete model takes into account electromechanical properties of MFC element including hysteresis and also properties of a voltage amplifier. The influence of the MCF model simplifications on the effective vibration control is presented.

Introduction

Rotating structures are of interest in many engineering applications. One of the classical examples are rotors of helicopters [1], drones or wind turbines. The blades of such rotors can be subjected to various loadings, for example can be excited by aerodynamic forces leading to flutter oscillations, and then large amplitude vibrations may arise. In order to avoid unwanted vibrations a special design of blades is proposed. The good possibility to obtain specific properties of the blade can be achieved by application of modern composite materials which enable creating specific mechanical features of the blade as presented in paper [2]. An additional option of a vibration reduction is to apply actuators embedded into the rotating blade which may reduce unwanted vibrations by use of dedicated control algorithms. The effectiveness of selected linear and nonlinear algorithms tested numerically and experimentally for a cantilever composite beam with active Macro Fibre Composite (MFC) elements was presented in paper [3]. The nonlinear saturation control and the positive position feedback control were demonstrated as the most powerful methods to suppress beam vibrations. However, the methods were tested for a fixed (non-rotating) structure. The proportional (P) and derivative (D) control method for the rotating beam was proposed in [4]. A pair of piezoelectric actuaturs/sensors were used to reduced vibrations of the structure. The analysis showed that typical P control method did not reduce vibration but only PD algorithm enhanced vibration damping of the rotating beam. The goal of this paper is to study dynamics of a rotating structure which takes into account more precise nonlinear model of the rotating beam (the plant) together with nonlinear properties of the emended MFC actuator. Then, the model will be used to propose and test a proper control algorithm, including linear and nonlinear control strategies.

Model formulation and results

A model of the rotating structure is composed of a rigid hub and a flexible blade oscillating in the rotating frame (x, y) which has position defined by a preset angle θ measured from the axis Z_0 of the rotating hub (Fig. 1a). A lumped mass m_t is added to the beam tip which allows to study more general case with dynamic boundary conditions. The beam with a rectangular cross-section is made of material having isotropic and linear properties. The beam is assumed to be thin and its model is based on Bernoulli–Euler beam theory. However, due to possible large deformations a nonlinear curvature and furthermore its extensionality is taken into account which, in case of rotating structure may play an important role. At the present study it is assumed that the hub rotates with constant angular speed $\dot{\psi}(t) = \Omega$ and, in addition, the beam is excited periodically by periodic loading distributed along the beam's spam.

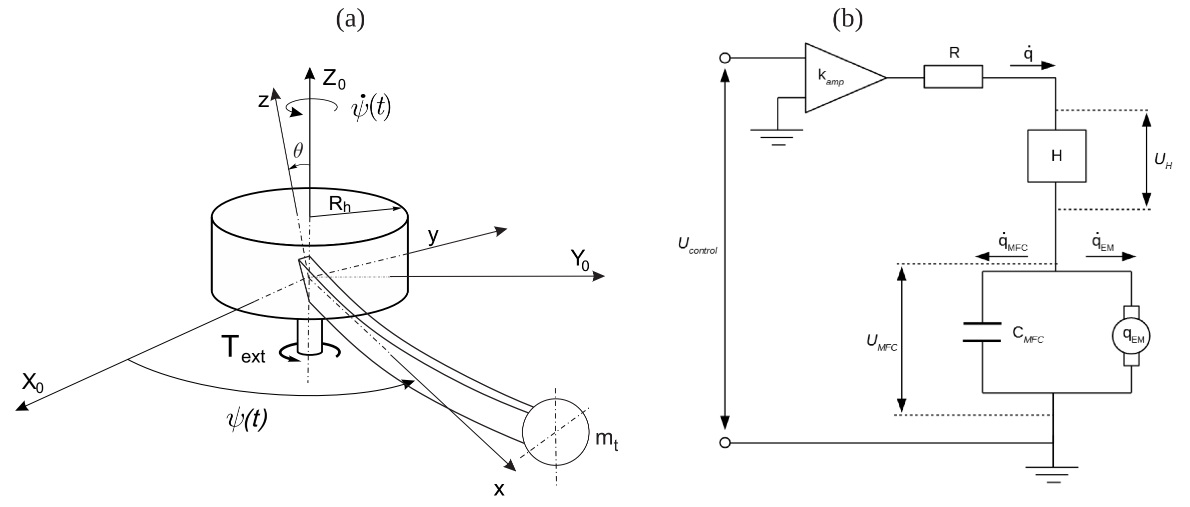

Figure 1: Model of the rotating hub–beam structure with tip mass (a) and schematic model of the MFC actuator (b).

The equations governing the beam dynamics are given as a set of partial differential equations (PDE) with associated dynamic boundary conditions. The equations have been solved analytically by the multiple time scale method. The resonance curves obtained analytically for the first and the second bending mode are presented in Fig. 2. The curves are computed for rotating structure for $\Omega = 10$ rad/s and selected preset angles $\theta = 30^0$, $\theta = 45^0$ and $\theta = 60^0$. The rotating beam demonstrates nonlinear beahaviour with hardening effect for the first mode (Fig. 2a) and softening for the second (Fig. 2b). The first resonance curve is sensitive for varied preset angle while the influence for the second is minor, almost invisible.

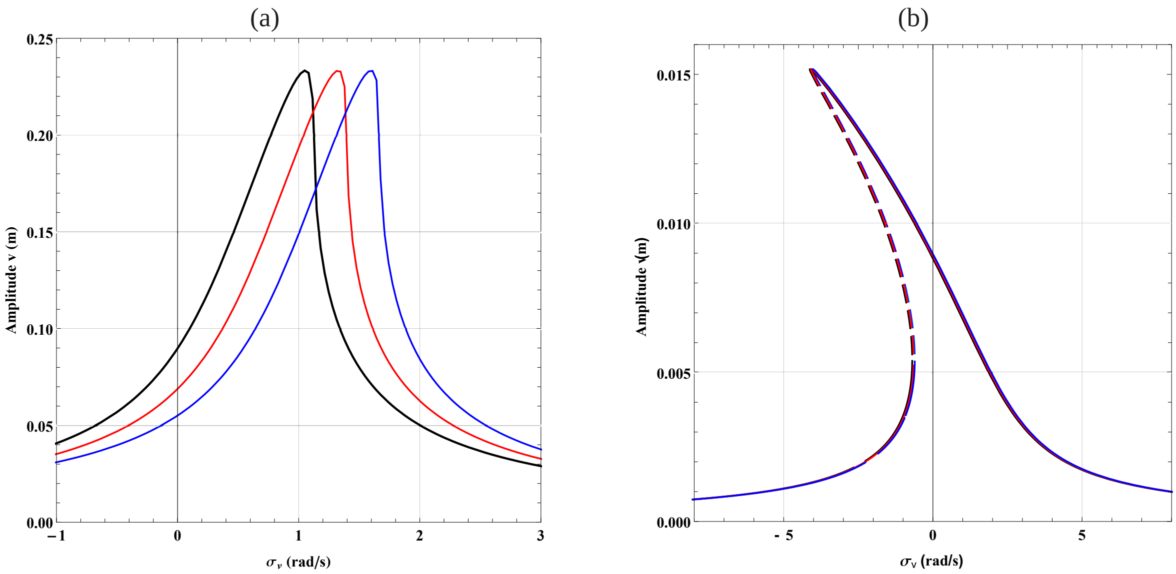

Figure 2: Resonance curves against frequency detuning parameter σ_v around the first (a) and the second (b) bending mode. Angular velocity $\Omega = 10$ rad/s and preset angle: $\theta = 30^0$ - black, $\theta = 45^0$ - red, $\theta = 60^0$ - blue.

To control the rotating beam and reduce vibrations the active MFC elements are embedded into the beam structure. The model of piezoelectric MFC element and its amplifier is presented schematically in Fig. 1b. In the scheme two elements k_{amp} and R describe amplifier properties. The gain k_{amp} is related to voltage $U_{control}$, resistor R models the output resistance of the amplifier. The piezoelectric actuator is represented by a capacitor with capacity C_{MFC}. The direct and indirect (converse) piezoelectric effect transform the actuator deformation into charge q_{EM} and actuator voltage U_{MFC} into mechanical force F_{EM}, respectively. The model also takes into account nonlinear properties of the piezoelectric actuator including the hysteresis phenomenon which schematically is represented by H element. The nonlinear and hysteretic piezo-element is added to the main structure (the plant) and the obtained combined electro-mechanical system is studied in order to select the most effective control algorithm enabling active vibration suppression for selected resonance states. Linear and nonlinear control strategies are tested and some of them are verified in laboratory.

Conclusions

The developed model of the nonlinear rotating beam which takes into account influence of angular velocity and preset angle is studied in the paper. Due to nonlinear geometrical nonlinearities hardening or softening effects are observed for the first and the second bending modes, respectively. It has been shown that varied preset angle affects mainly the first mode resonance observed by a shift of the resonance zone. The second mode resonance is almost not affected, just a very minor change is present. Apart from the nonlinear rotating beam model (the plant) also the model of active MFC element is proposed. The model takes into account two-way electromechanical coupling as well as hysteresis of the active element. The control strategy for active vibration damping with linear and nonlinear control algorithms is analysed.

Acknowledgements
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