Modeling Nonlinearities in MEMS Micro Mirrors: From Single Chip to Wafer Level

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Summary. The performance requirements for MEMS sensors and actuators, such as scanning micro mirrors, are increasing as emerging applications in the fields of highly-automated driving or augmented reality are in increasing demand. During final testing, a device is rejected as soon as it does not fulfil the performance specifications which decreases yield and thus increases the overall cost. In many cases, unexpected device failure or performance issues can be traced back to nonlinear system behaviour. This entails the need for system models which take nonlinear sensor dynamics into account and a fundamental understanding of the underlying nonlinear physics is often essential for an improved MEMS design. Here, we present a comprehensive analysis of nonlinear dynamics in scanning MEMS micro mirrors ranging from careful measurements and modeling of nonlinear system behaviour on the level of individual chips up to wafer-level testing of several hundred devices. The underlying nonlinear mode-coupling phenomenon, known as spontaneous parametric down-conversion (SPDC) exhibits a sudden transition from mostly linear to nonlinear system behaviour. The threshold amplitude or rather critical point only lies within the operational amplitude of a device, when a specific frequency resonance condition (a 1:1:1 internal resonance) for the mechanical modes of the device is closely matched. Due to fabrication imperfections of MEMS process technologies small deviations in the geometry between several devices of the same design occur which consequently influences the frequency spectrum strongly influence the frequency spectrum of each individual device and therefore decide about the fulfilment of the resonance condition. We demonstrate the benefits that can be achieved by employing the insights gained from single-chip measurements and models to the analysis of a large number of devices on wafer level and suggest a possible path towards successful design iterations. Moreover, above the threshold we show that the micro mirror displays fundamental nonlinear behavior ranging from stationary state bifurcations to dynamical instabilities.

Introduction

One main requirement for scanning MEMS micro mirrors for automotive as well as for consumer applications is the reachable deflection angle. Yet, our experimental observations show that even micro mirrors of the same design layout often show drastic differences in their maximum deflection [3]: Some devices reach the required deflection angle while others exhibit instabilities in their deflection and sometimes even fracture.

These drastic differences in the device behaviour can be attributed to small differences in the mode spectrum caused by geometry variations due to the process tolerances of surface micromachining. In the case of the scanning MEMS micro mirror at hand, the nonlinear dynamic behaviour which occurs in some cases is due to a fundamental nonlinear model, more prominently known from nonlinear optics, namely spontaneous parametric down-conversion (SPDC). In the mechanical domain, where the nonlinear mode coupling originates from geometric nonlinearities, SPDC or three-wave mixing has only been studied in recent years [1, 3].

For SPDC, a model of an externally driven, damped Duffing oscillator with a cubic nonlinearity is used for the drive mode. Additionally, two so-called parasitic modes of the same type are coupled to the drive mode through a three-wave coupling term. In addition, as discussed in [2], a nonlinear damping term needs to be included.

Full Model for Single Chips

With the a simple model of three nonlinearly coupled ordinary differential equations, the observed effects can be simulated and thus explained physically. They include bifurcations and hysteresis, as well as unstable behaviour caused by a resonant actuation of parasitic modes and thus amplitude depletion of the drive mode. Additionally, critical slowing-down and limit cycles can explain the observed phenomena.

The system parameters needed to emulate the behaviour can be measured directly using laser doppler vibrometry. They can also be obtained from simple optical measurements, where the deflection angle is measured during the undisturbed and during the disturbed operation of the mirror.

Fig. 1 shows the modelled behaviour of a micro mirror that shows SPDC behaviour. As mentioned above, SPDC is a spontaneous process and thus, the figure shows the behaviour below threshold (identical to the nonlinearly damped Duffing oscillator) as well as the behaviour above threshold, where the two parasitic modes gain amplitude and disturb the oscillation of the drive mode. The critical point model represents the threshold deflection angle, that is the transition from uncoupled behaviour to SPDC behaviour with the two additional parasitic modes. This model is of vital importance for the large scale application and wafer level testing that will be elaborated on in the next section.

Critical Point Model for Wafer Level

The simple analytical model for the critical deflection angles that was shown as a red line in Fig. 1, becomes relevant when measurements need to be carried out for a larger number of devices in order to assess their nonlinear behaviour.

Thus, it is especially important for engineering applications, since it allows for a differentiation between devices that lie within the specifications and devices that lie outside
As we show in [3], mode coupling can be predicted with low computational effort. The result of the prediction only includes at what angle mode coupling occurs in a specific mirror and does not characterize the full range of nonlinear dynamic effects as was the case for the full model above. Yet, this prediction is sufficient for the decision on whether or not a device meets the requirements pertaining to the deflection angle. It points out the individual chips with critical deflection angles below a certain threshold. Apart from this, it increases knowledge about the design and thus provides input for design iterations and optimization.
Fig. 2 show the results of the critical point model [4]. The design parameter $\delta = f_{0,d} - f_{0,p_1} - f_{0,p_2}$ on the horizontal axis gives the relation of the linear mode frequencies of the three coupled modes, where $f_{0,0}$ denotes the frequency of the drive mode and f_{0,p_1} and f_{0,p_2} the frequencies of the two parasitic modes. In other words, δ represent the 1:1:1 internal resonance condition.

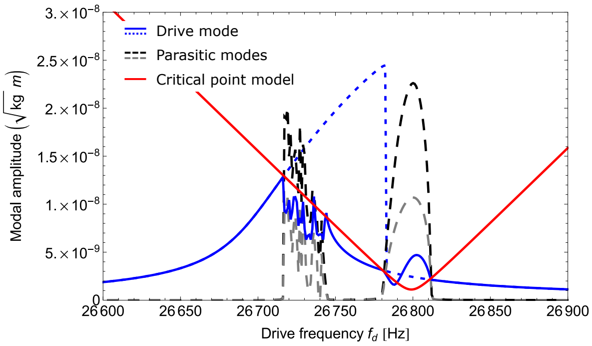

Figure 1: Transient representation of a sweep with increasing frequency. The drive mode is shown in blue, for the case of mode coupling, the line is solid. For comparison, the dotted blue line represents the behaviour of the drive mode without any mode coupling. The dashed lines in grey and black show the two parasitic modes during mode coupling. The red line denotes the critical point model which depicts the onset of SPDC and will be of great importance in the following section.

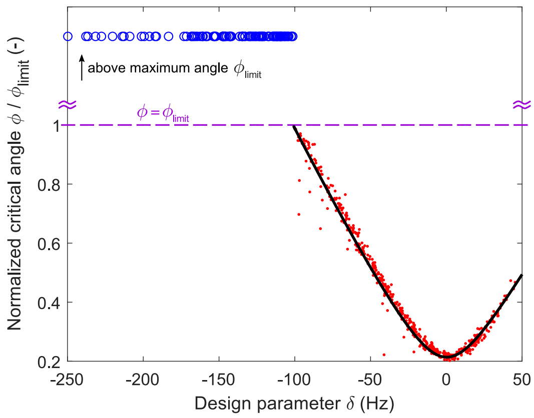

Figure 2: Validation of the critical point model: The measured critical deflection angles (631 points) which mark the onset of mode coupling, where red dots and blue circles distinguish critical angles below and above the specified deflection angle, are compared to the prediction from the critical point model, shown as a black line.

Conclusion

We showed that when the modes of a mechanical structure fulfil an internal resonance condition and show a large coupling strength, nonlinear phenomena such as SPDC can occur. The behaviour of single chips was modelled using the full model for three coupled equations of motion with the relevant nonlinear terms from structural mechanics. Wafer level testing of the critical threshold for the onset of mode coupling was successfully matched with the prediction of the much simpler critical point model. The design parameter δ provides leverage for adjusting the mode coupling of a MEMS design. Thus, we explained both the underlying nonlinear dynamics effects and their implications for unwanted nonlinear system behaviour. Connecting these two parts is highly relevant for the development of not only MEMS micro mirrors, but any resonant MEMS sensor or actuator. A range of similar coupling mechanisms due to geometric nonlinearities is also conceivable.

References

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