Parametric and Self-excitation of a Suspension Bridge under Turbulent Wind Flow

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Summary. In this paper, the analysis of a self-excited suspension bridge under turbulent wind flow is carried out. The stationary wind is responsible for self-excitation, while the turbulent part is responsible for parametric excitations. The simultaneous presence of those excitations is taken into account in a specific resonance condition. The periodic solutions are studied by means of a perturbation method and the effects of the turbulence on the dynamics of the structure are analyzed.

Introduction

Suspension bridges are long, slender flexible structures which are very sensitive to dynamic actions induced by wind, which causes a variety of instability phenomena, related to different kind of excitation. In particular, the aeroelastic instability dealing self-excited vibrations, such as galloping and flutter, is of particular interest, since these phenomena may cause devastating effects, leading to structural collapse like in the famous Tacoma Narrows bridge. The aeroelastic behavior of long-span bridges, and especially the aeroelastic instability, has drawn remarkable attention in the fields of structural engineering and physics. The modern era of bridge aeroelasticity was launched by [1]-[4]. More recently, many linear and nonlinear aeroelastic analysis frameworks for cable-supported bridges were developed, e.g. [5]-[9], to predict the aeroelastic response of bridges under steady and turbulent winds (the most by solving the motion equations by numerical tools).
In this paper, the aeroelastic behavior of a suspension bridge is investigated by a continuous model. A simple analytical model of suspension bridge, subjected to turbulent wind flow, is proposed to analyze the aeroelastic in-plain instability (galloping). The objective is to take into account the possible occurrence of Hopf bifurcations, due to the steady part of the wind, and to analyze modifications on the solutions due to the turbulent part. The main innovative aspect relies in directly attacking the continuous problem by perturbation methods, leading to approximate formulae suitable for preliminary designs.

Model

A standard single-span suspension bridge, made of a main cable, a stiffening girder, uniformly distributed hangers (or suspenders) and two supported towers or pylons, is considered. The main cable is rigidly connected to the support towers, assumed to be rigid. The main cable is assumed to be uniform and elastic, and its bending stiffness is ignored. The structure is subjected to a turbulent wind flow of velocity $U(t)$, blowing orthogonally to the plane of the bridge. A scheme of the model is shown in Fig. 1.

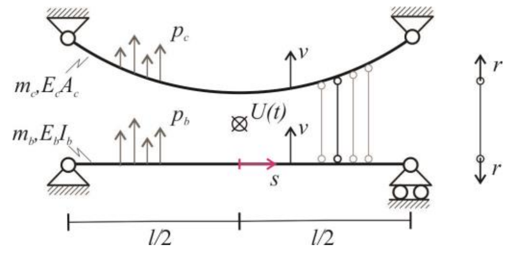

Figure 1: Single-span suspension bridge model.

Linear visco-elastic continuous models of beam and cable are formulated, coupled by vertical suspenders, modeled as uniformly distributed axially rigid links. Both external and internal damping are accounted, this latter according to the Kelvin-Voigt rheological model. The aeroelastic effects of the wind are evaluated via the classical quasi-static theory.

Aerodynamic forces
The aerodynamic load is caused by the wind, which blows orthogonally to the plane of the bridge, with time-dependent velocity $U(t)$. This, triggers in-plane forces, which depend on the structural velocity, as result of the aeroelastic interaction. Only forces on beam are accounted, while force on the cable are neglected. Nonlinear aerodynamic forces are formulated in the framework of the quasi-steady theory (see [10]); by truncating them to the third-order, they read:

$$p^a = -\left(b_1 U(t)\dot{v} + b_3 \frac{1}{U(t)}\dot{v}^3\right) \tag{1}$$

where the coefficients b_i are aerodynamic coefficients depending on the shape of the cylinder cross-section. The wind velocity is decomposed as $U(t) = \bar{u} + u(t)$, where $\bar{u}$ is a constant (average) part, representing the steady component, and $u(t)$ is a periodically time-dependent part, representing the turbulence. By assuming that the turbulent part is small compared to the steady one, the aerodynamic force is expanded is Taylor series.

Equation of motion

The dimensionless motion equations read:

$$
\begin{aligned}
&\rho^2(1+\eta_b\partial_t)v'''' - (1+\eta_c\partial_t)v'' + \Lambda^2(1+\eta_c\partial_t)\int_{-\frac{1}{2}}^{\frac{1}{2}} v\,ds + \ddot{v} + \left(c_e + b_1\left(\bar{u}+u(t)\right)\right)\dot{v} + b_3\left(\frac{1}{\bar{u}} - \frac{u(t)}{\bar{u}^2}\right)\dot{v}^3 = 0\\
&v\left(-\tfrac{1}{2},t\right) = 0 \quad \rho^2(1+\eta_b\partial_t)v''\left(-\tfrac{1}{2},t\right) = 0\\
&v\left(\tfrac{1}{2},t\right) = 0, \quad \rho^2(1+\eta_b\partial_t)v''\left(\tfrac{1}{2},t\right) = 0
\end{aligned}
\qquad (2)
$$

The dimensionless parameters Λ^2 and ρ^2 accounts: the first, for the elastic and geometric properties of the cable (known as the Irvine-Caughey cable parameter [11]); the second, for the beam-cable stiffness ratio. The dimensionless terms η_b, η_c and c_e are internal and external damping parameters. The Eqs. (2) are a generalization of the motion equations provided for the first time by Bleich et al. in [12], and recently re-examined by [13], accounting for additional damping and aerodynamic loads. They describe an infinite-dimensional system, parametrically excited by the turbulence. Here the turbulent part is considered harmonic, i.e., $u(t) = \hat{u}\cos(\Omega t)$, where $\hat{u}$ is the amplitude and $\Omega = 2\omega + \sigma$ is the frequency of the parametric excitation, chosen close to the double of the natural frequency ω, with σ a small detuning. Therefore, the condition of principal parametric excitation occurs.

To investigate the behavior of the structure in the nonlinear field, close to the dynamic bifurcation, a nonlinear asymptotic analysis is carried out. The Multiple Scale Method (MSM) is used, by directly attacking the partial differential equations of motion. A bifurcation equation in the amplitude of motion $a(t)$ is obtained, from which periodic oscillations and their stability are analyzed.

Main results

The mains results consist in the stability domains (Fig. 2a) and bifurcation diagrams (Fig. 2b).

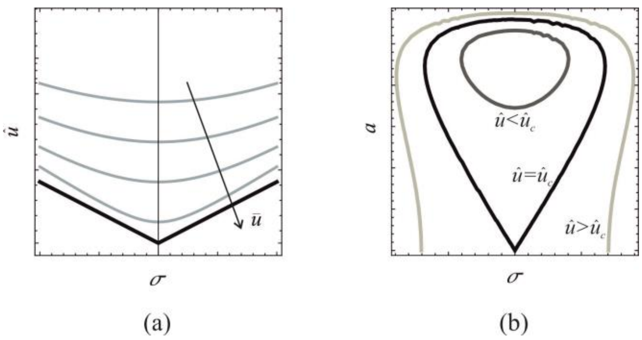

Figure 2: (a) Stability domains; (b) Bifurcation diagrams in presence of turbulent wind.

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