Resonance frequency measurement of stress-engineered nanomechanical resonator and its lower limit of frequency uncertainty

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Summary. In this work, we experimentally measure a relative frequency stability below 10^{-6} for a stress-engineered ≈ 28 MHz nanomechanical resonator by detecting and analyzing its thermodynamic fluctuations at room temperature, without external excitation. We devive the Cramer-Rao lower bound (CRLB) thermodynamic limit for resonance frequency measurement precision for a classical harmonic oscillator subject to dissipation, thermodynamic noise, detection uncertainty and with or without external excitation. We propose a general statistically efficient frequency estimator and experimentally show frequency uncertainty reaching the CRLB on the cavity-optomechanically detected ≈ 1 pg resonator data for up to ≈ 0.1 s averaging. The stress-engineered nanomechanical resonator with high frequency-Quality factor (fQ ≈ 10^{12}) and considerable frequency stability could be used for frequency-readout displacement sensors with no excitation required.

Introduction and Novelty

The work is related to an earlier report about passive frequency stabilization [1] which uses the same type of high fQ tuning fork resonator [4]. Resonance frequency variance is critical to the performance of nanoresonators. With sufficient motion detection precision, frequency measurement is fundamentally limited by the thermodynamic fluctuations. Lower measured frequency variance is achieved by increasing the driven amplitude. However, as the thermodynamic uncertainty is lowered, the variance becomes limited by frequency drift at ever shorter time scales. Here, we study the frequency variance of a stress-engineered nanomechanical resonator with an effective mode mass of ≈ 1 pg. First, without external excitation, the relative frequency stability is reaching below 10^{-6}, which is better than the average performance of state-of-the-art driven NEMS in such mass range [2]. Second, more importantly, we derive rigorous CRLB, establishing the theoretically lowest limit for the resonance frequency estimation variance. It is applicable to any linear harmonic resonator, including the Micro- and Nano-electron-mechanical systems. Our undriven and driven devices perform at those limits up to the averaging times of ≈ 0.1 s without any extra stabilization [3]. Finally, we present a computationally-fast and statistically efficient frequency estimator–a formula for converting motion records into frequencies with imprecisions at the CRLB. The presented general analysis is applicable to any undriven or harmonically-driven M/NEMs in the linear regime and may be extendable to the nonlinear regime by considering a nonlinear equation of motion, such as a duffing oscillator.

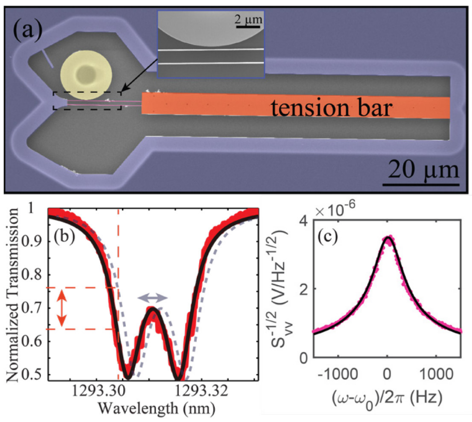

Figure 1: Experimental system introduction. (a) False-colored SEM image of the stress-engineered nanomechanical resonator. (b) Measured optical resonance and the operating principle of the optomechanical detection. (c) Mechanical vibration power spectral density in a vacuum.

Results and Discussion

In this work, stress engineering is applied to increase fQ products of nanoscale Si_3N_4 tuning forks, reaching 10^{12} to 10^{13} range. A false-colored SEM image of the device with a stress tuning bar is shown in Fig. 1(a). The mechanical motion of the tuning fork modulates the optical resonance frequency of an adjacent microdisk resonator (Fig. 1(b)), generating time-varying optical transmission signals proportional to mechanical displacement. Fig 1(c) shows the mechanical power spectral density of the thermodynamically fluctuating tuning fork mode.

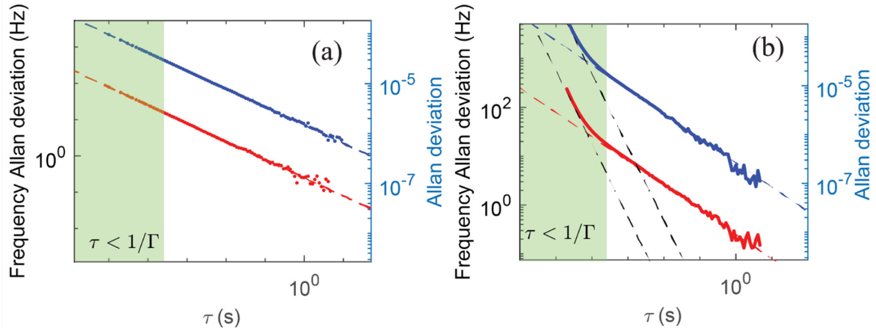

Figure 2: ADEV for simulated data(solid lines) with (a) negligible motion detection noise and (b) experimentally-realistic detection noise added. ADEV without (blue) and with (red) external excitation agrees with theoretical CRLB (dashed lines)

To verify the validity of the derived CRLB and the efficiency of the derived frequency estimator, we first apply the estimator to a simulated motion trace of a fluctuating resonator and calculate the resulting frequency Allan deviation (ADEV). Following the expected $\propto \tau^{-1/2}$, the ADEV quantitaively agrees with the CRLB without any adjustable parameters on all time scale for both the driven and non-driven cases, provided detection noise is negligible (Fig. 2(a)). The maximum-likelihood frequency estimator includes the contributions from both the phase and the phase derivative for the frequency estimation. When artificial motion-detection noise is added to the simulation, the ADEV becomes detection noise limited at the shortest time scales (widest bandwidth), following $\propto \tau^{-3/2}$. The ADEV follows $\propto \tau^{-1/2}$ and quantitatively agrees with the CRLB for the longer time scales, where our CRLB is valid (Fig. 2(b)).

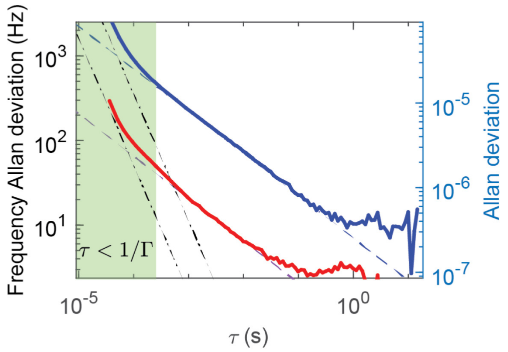

Figure 3: ADEV (solid lines) for experimental data without (blue) and with (red) external excitation. Dashed lines are corresponding theoretical CRLB $\propto \tau^{-1/2}$. Dot-dash lines $\propto \tau^{-3/2}$ are the theoretically expected *limits from the detection noise.*

The ADEV for experimental data presented in Fig. 3 shows similar features to the simulated data with detection noise, and quantitatively agrees with CRLB without adjustable parameters for $\tau < 0.1$ s. At longer averaging the resonance frequency drift becomes apparent. Remarkably, the resonator without any applied excitation (blue line) shows a low frequency ADEV at the thermodynamic limit of ≈ 7 Hz/Hz$^{0.5}$ below 1 s and (relative) stability of ≈ 10 Hz (≈ 0.40×10^{-6}) above 1 s averaging. The undriven and driven devices perform at CRLB up to averaging times of about 1 s and 0.1 s.

Conclusions

In conclusion, we present a statistically efficient resonance frequency estimator and the thermodynamic limit of resonance frequency measurement uncertainty applicable to linear resonator subjects to harmonic drive, thermal fluctuations, and detection uncertainty, on all time scales. After validating them with simulated data, we have implemented the frequency estimator on a stress-engineered nanomechanical resonator. Remarkably, the resonator remained at the thermodynamic limit even for fairly long averaging times, and better than part per million frequency stability has been measured, even though the resonator was driven solely by thermal fluctuations. The stress-engineered nanomechanical resonator with high frequency-Quality factor and considerable frequency stability may be used for developing frequency-readout displacement sensors requiring no excitation.

References

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