Finite time bias removal in multi-agent non-linear systems*

V.S. Varma*, S. Srikant† and I.-C. Morărescu*
*Université de Lorraine, CNRS, CRAN, F-54000 Nancy, France,
†IIT Bombay, India.

Summary. This work proposes some iterative procedures for estimating additive measurement biases in multi-agent non-linear systems. The bias is a constant additive offset in the relative measurement of states between agents. In contrast to existing literature which estimate biases asymptotically or does adaptive control to remove the impact of bias, we provide an iterative procedure that estimates the bias in a finite number of steps in the noiseless case.

Introduction

The multi-agent formalism enables treating problems that arise in many application domains such as engineering [3], sociology [5] or biology [9]. Consensus and synchronization were mainly studied for agents with linear dynamics interacting through a graph, see [6] for instance. However, there are also studies with nonlinear dynamics, see for example [8] which considers oscillators dynamics, or [3] on non-holonomic robots. However, the effect of measurement biases have not been well explored in this context. In [11], the authors develop an adaptive control to correct biases in a double integrator but it is not clear how this method can be adapted to general non-linear systems. In [2] and [7], the authors proposed algorithms to estimate sensor offsets in wireless sensor networks. These methods only partially compensate the offsets and never completely eliminate the bias. Recently, in [10], the authors propose a distributed bias removal strategy using a consensus type algorithm, however, the convergence of the estimate to the true bias is asymptotic.
Our main contribution is to provide a method of bias *removal in finite time* when dealing with nonlinear multi-agent systems subject to constant bias measurements. The method proposed here requires a limited number of communication instances and can be implemented either in a centralized manner through *cloud computing* or in a decentralized manner as long as *two interacting agents can measure the same reference point.*
Problem formulation: We consider a network of n interacting agents, where the interactions are described by a graph $G = (V, E)$. Each agent $i \in V$ is described by a state $x_i \in \mathbb{R}^{n_x}$. $\mathcal{N}_i = \{j \in V \mid (i, j) \in E\}$ specifies the neighborhood of agent i and represents the set of neighbors whose relative states can be measured by i. However, the measurements made by any agent i have a constant additive bias b_i which must be removed in order to accomplish the overall common goal. For any $j \in \mathcal{N}_i$, agent i has access to the measurement $z_{i,j}(t) = x_j(t) - x_i(t) + b_i$ and implements a distributed control $g(z_i)$ in order to achieve a cooperative task.
The agent dynamics is given by

$$x_i(t + 1) = f_i(x_i(t), g(z_{i,j})). \tag{1}$$

We assume that the above algorithm performs well when the additive bias is 0. However, the presence of the additive bias leads to significant deterioration in performance and this must be corrected by taking $\bar{z}_{i,j} = z_{i,j} - \hat{b}_i(t)$, with $\hat{b}_i(t)$ assumed to be an estimate of the bias. Our objective is to estimate b_i for all i in finite time, using communication with a cloud or only among neighboring agents when a reference can be measured.

MAIN RESULTS

Estimating the additive biases without communication with neighbors is not possible since all the measurements have the same bias and this parameter can not be removed. In the following, we provide the assumptions on communication and on the graph structure so that bias can be estimated in a finite number of steps . .

Bias removal for non-bipartite connected graphs

Assumption 1 *We look at connected, undirected and non-bipartite graphs and assume that all agents can identify the tag of their neighbors and themselves.*

We use L^+ to denote the sign-less Laplacian which can be defined as having $L^+_{ij} = 1$ when $j \in N_i$, $L^+_{ij} = |N_i|$ if $i = j$ and 0 otherwise.

Lemma 1 ([4]) *The matrix L^+ is invertible when the graph is non-bipartite.*

We assume that all agents can communicate with the cloud which will then process all the information it has to estimate the bias. In the proposed procedure, all agents i will measure $z_{i,j}(0)$ for all its neighbors j and communicate this information to the cloud (synchronously or asynchronously). Once all agents have communicated their measurements to the cloud, it can use the following result to estimate all their biases.

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Proposition 1 *Under Assumption 1, the cloud can estimate the bias for all agents as* $\hat{b} = (L^+)^{-1}\delta$ *where*

$$\delta_i = \sum_{j \in N_i} z_{i,j} + z_{j,i}. \tag{2}$$

From [1], we know that all non-bipartite graphs contain an odd-ring as a sub-graph. So the above procedure can work as long as the graph contains an odd-ring. In the absence of a cloud/central entity, the following distributed procedure may be used.

Remark 1 *When there is no central-cloud, if all agents have sufficient memory and can communicate with their neighbors, then all agents can estimate their bias in* N *steps. At each step all agents will communicate all the new information they have collected in the last step allowing them to aggregate all the information required to apply proposition 1.*

Distributed bias estimation with a reference

Assumption 2 *The graph is connected and there exist at least two neighboring agents* i, j *who can measure a common reference* r*.*

We use $z_{i,r}$ to denote the relative measurement of the reference with respect to agent i. In this case, we propose a distributed procedure for bias estimation in finite time. First, all agents i communicate $z_{i,j}(0)$ and $z_{i,r}(0)$ (if the reference can be measured by i) to all its neighbors.

Proposition 2 *Under Assumption 2, at least one agent* i *will be able to estimate*

$$\hat{b}_i = z_{i,r}(0) - (z_{j,r}(0) - z_{j,i}(0)). \tag{3}$$

after it receives communication from its neighbor j *which can also measure the reference.*

We propose the following distributed procedure for bias estimation in finite steps (at most N) for the remaining agents. The algorithm applied by any agent i can be described as follows.

Data: At time 0, measure and communicate $z_{i,j}(0)$ and $z_{i,r}(0)$ (if available) to all neighbors j.
if $z_{j,r}(0)$ *and* $z_{i,r}(0)$ *are known after receiving communication from neighbor* j **then**
  estimate $\hat{b}_i$ using proposition 2.
else
  $d_{i,j} \leftarrow z_{i,j}(0) + z_{j,i}(0)$
end
while $\hat{b}_i$ *is not estimated* **do**
  Wait for transmissions ;
  if *some neighbor* j *transmits* $\hat{b}_j$ **then**
    Estimate $\hat{b}_i \leftarrow d_{i,j} - \hat{b}_j$
  end
end
Transmit $\hat{b}_i$ to all neighbors.

Algorithm 1: Procedure of bias estimation

Since the graph is connected due to Assumption 2, the iterative procedure described above will have all agents estimating their own biases within N steps at most.

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