Non-linear dynamics of straight beams with (or without) shape imperfections and very shallow arcs: similarities and differences controlled by boundary conditions

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Summary. The free and forced vibration problems for slightly curved axially restrained beams are solved in two alternative formulations. The first one is the classical nonlinear model, which accounts for the so-called 'stretching-due-to-bending' effect. In its framework, a slightly curved beam is considered as the straight one with an initial shape imperfection. The second formulation is the standard model of a planar curved beam. Each model describes the interaction between transverse and longitudinal motion of a beam, but in profoundly different manner. Comparison of predictions of these models for small shape imperfections and various boundary conditions is the goal of this paper.

Introduction

Nonlinear vibrations of ideally straight axially restrained hinged beams have been studied in numerous publications with [1-3] being the most recent ones. The canonical model of such a beam implies that an interaction between flexural and axial vibrations occurs due to the essentially non-linear 'stretching-due-to-bending' effect. This coupling mechanism is preserved when a beam acquires arbitrarily small initial shape imperfections. However, vibrations of a slightly curved axially restrained beam can conveniently be modelled within the standard linear theory of planar curved beams, which captures an interaction between flexural and axial vibrations regardless the type of boundary conditions. Furthermore, the nonlinear theory of dynamics of planar curved beams is also well established [4-5]. In this framework, an obvious research question arises:

- What are the validity ranges of the non-linear theory of ideally straight axially restrained beams, the linear theory of curved beams and the non-linear theory of curved beams with respect to the amplitude of initial shape imperfection (initial curvature), vibration amplitudes and boundary conditions?

An attempt to find answer to this question constitutes the subject of the work in progress. Its first preliminary results are presented in what follows.

The model of a curved beam

Our point of departure is the nonlinear model of a circular cylindrical shell [4] adjusted to the plane strain state of deformation. Then two components of displacement (u, w) are considered and the coordinate **s** along the curved axis of a shell's segment (which is now reduced to a beam) is introduced, see Figure 1.

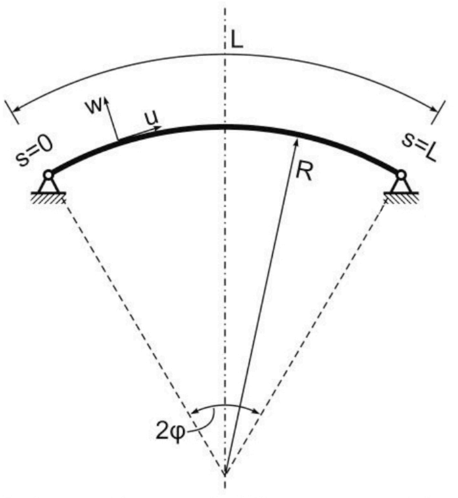

Figure 1: A curved beam with sliding support and force applied

The governing non-linear differential equations obtained by means of the Hamilton's principle are ($\beta \equiv \frac{L}{R}$):

$$-\frac{\rho L^2\left(1-\nu^2\right)}{E}\ddot{u}+u''+\beta w'+\left(w'-\beta u\right)\left(w''-\beta u'\right)+\frac{h^2}{12L^2}\beta\left(\beta u''-w'''\right)$$
$$+\left[u'+\beta w+\frac{1}{2}\left(w'-\beta u\right)^2\right]\left(w'-\beta u\right)\beta=0$$
$$-\frac{\rho L^2\left(1-\nu^2\right)}{E}\ddot{w}-\left[u'+\beta w+\frac{1}{2}\left(w'-\beta u\right)^2\right]\beta+\frac{h^2}{12L^2}\left(\beta u'''-w''''\right)$$
$$+\left[u''+\beta w'+\left(w'-\beta u\right)\left(w''-\beta u'\right)\right]\left(w'-\beta u\right)+\left[u'+\beta w+\frac{1}{2}\left(w'-\beta u\right)^2\right]\left(w''-\beta u'\right)=0$$

The boundary conditions at s=0 are $u=w=\beta u'-w''=0$, while the boundary conditions at s=1 for the case illustrated in Figure 1 are

$$u'+\beta w+\frac{1}{2}\left(w'-\beta u\right)^2+\frac{h^2}{12L^2}\left(\beta u'-w''\right)\beta+\frac{p\left(1-\nu^2\right)}{Eh}\cos\left(\frac{L}{2R}\right)=0$$
$$\left[u'+\beta w+\frac{1}{2}\left(w'-\beta u\right)^2\right]\left(w'-\beta u\right)+\frac{h^2}{12L^2}\left(\beta u''-w'''\right)+\frac{p\left(1-\nu^2\right)}{Eh}\sin\left(\frac{L}{2R}\right)=0$$
$$\beta u'-w''=0$$

Alternative formulations of boundary conditions at s=1 (e.g., sliding support in an absence of the force or full constrain of displacements) are obvious. Likewise, the limit cases of a linear curved beam [4-5] and a nonlinear ideally straight beam [1-3] are readily available from the proposed formulation.

The solution methods and the eigenfrequency analysis

The exact solution of the linear problem of free/forced vibrations of a curved beam is easily obtained regardless the type of boundary conditions. To solve a non-linear problem of vibrations of a curved beam, the canonical method of multiple scales adjusted for the system of partial differential equations is used. The reference solutions of a nonlinear problem of vibrations of an axially restrained beam by the same method are presented in the references [1-3].

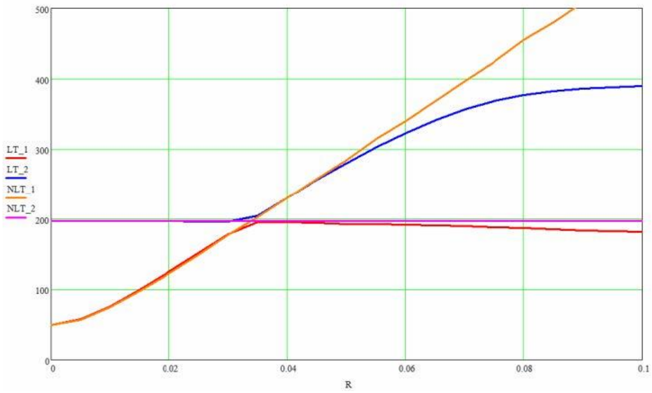

Figure 2: The first and the second eigenfrequency of a curved beam versus curvature parameter

The results of eigenfrequency analysis of an axially restrained hinged beam are presented in Figure 2. The appreciable (exceeding 5%) differences correspond to the amplitude of shape imperfection around 0.06L.

Conclusions

The results obtained so far suggest that the simple linear theory of curved axially restrained beams agrees with the nonlinear theory of straight beams with shape imperfections in a broad range of curvature parameter. Nonlinear vibrations of a curved beam still require careful asymptotic analysis, especially in the case of a vanishingly small curvature.

References

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