A novel lubricated friction model for describing underdamped free responses of a spring – sliding mass oscillator

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Summary. A novel friction model is introduced in order to describe lubricated sliding contact under unsteady or transient dynamic conditions. It is based on a state variable friction model consisting on a friction force equation and a state ones. Effective film thickness is introduced as the internal variable. Relaxation behaviour to reach the steady state film thickness is introduced via a first order differential equation. To obtain friction force, film thickness is interpreted as a sharing effect between solid interaction or confined lubricant and the lubricant film itself. Experimental results consisting on free dynamic responses of a sliding oscillating system under lubrication confirm the phenomenon captured by the proposed theoretical approach.

Introduction

Friction is a nonlinear phenomenon that occurs in a lot of mechanical systems including contacts. This is in particular the case of mechanisms which are often making use of contacting surfaces in relative motion (gears, cam and cam follower systems, rolling bearings, piston ring/cylinder liner contact, universal joints, etc.) for transforming input forces and motion. In order to simulate and analyze the dynamic behavior of such systems, it is necessary to choose friction law that can be able to describe and model it. A classical way to describe friction is to use the Coulomb friction or viscous friction models, or the combination of both [1]. Such friction models are very useful thanks to their simplicity. However, in many circumstances, they cannot accurately describe tribological behaviors. In the case of lubricated contacts, other models have been used to reproduce the well-known Stribeck behavior, but they usually ignore the dynamic effects [2]. In this context, the principal aim of this study is to introduce a novel approach which can describe lubrication in a relative simple way, and capture the dynamic lag effect of the film thickness settling [3]. Finally, comparisons between numerical and experimental results obtained from an original dynamic tribometer has been reported in this study.

The suggested friction model

The proposed model is based on introduction of an internal state variables in the constitutive friction law. In this approach, modelling consists on a friction force equation coupled to a state equation, which can be formally written as

$$\begin{cases} T = & \mathcal{F}(v,y) \\ \dot{y} = & \mathcal{G}(v,y) \end{cases} \tag{1}$$

where y is the internal state variable and $\mathcal{F}(\cdot)$ and $\mathcal{G}(\cdot)$ are two nonlinear functions to be precise. The well-known rate-and-state-variable friction law [4] which is formulated in term of an average contact lifetime as the internal state variable is a good example of this kind of formulation. In our novel model, we heuristically suggest to introduce the instantaneous film thickness as the internal state variable y. Indeed, this is the key to interpret transition between boundary and full film hydrodynamic regimes revealed by the popular Stribeck curve [5]. For the state equation, we propose a simple relaxation process governed by the following differential equation

$$\dot{y} = (Y_{ss}(v) - y)/\tau \tag{2}$$

where $Y_{ss}(v)$ represents the film thickness to be reached. In this way, it corresponds to the steady state film thickness associated to the instantaneous sliding velocity $v(t)$. In addition, τ is the relaxation time associated to the exponential decay introduced by the equation (2). Concerning the friction equation, we introduced a mixed law given by

$$T = (1-\alpha)T_{BL} + \alpha T_{HL} \tag{3}$$

Thus, the total friction force is assumed to be sharing between friction force T_{HL} associated to the lubricant film itself and friction force T_{BL} associated to solid interaction through surface roughness. In agreement with the Stribeck curve interpretation, the weight function $\alpha = \alpha(y)$ introduces the effect of the film thickness y. Basically, it consists on a sigmoid function which increases monotonically from 0 to 1. Finally, the friction force component T_{BL} is assumed to be constant, i.e. $T_{BL} = \mu N \mathrm{sign}(v)$, and the friction force component T_{HL} is assumed to be velocity-dependent through a power law $T_{HL} = \eta N |v|^p \mathrm{sign}(v)$. N represents the applied normal load and μ, η are two constants.

The dynamic transient oscillating response under study

As reported in previous works [1,6,7], we have designed and built a dynamic tribometer in order to quantify general trends of friction as a function of the sliding velocity. This experimental setup, described in Figure 1, is based on the measurement of transient responses $x(t)$ of an underdamped frictional single-degree-of-freedom mass-spring oscillator. In the principle, a

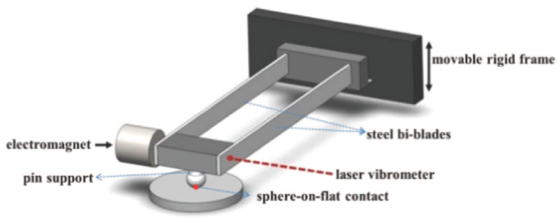

Figure 1: Scheme of the dynamic oscillating tribometer

moving mass m slides in contact with the tribological system under study.
The governing equation of motion can be written as follows

$$m\ddot{x} + kx = -T \quad (4)$$

where k is the spring stiffness. By considering the proposed friction model, it follows the first order equations

$$\begin{cases} \dot{x} = & v \\ \dot{v} = & -kx - T(v, y) \\ \dot{y} = & (Y_{ss}(v) - y)/\tau \end{cases} \quad (5)$$

with the friction force $T(v, y)$ defined by equation (3). In order to integrate equations (5), we have used a Runge Kutta numerical scheme of order 4.

Results

Figure 2 (a) shows example of an experimental velocity response. We can observe that it begins with a convex envelop and finish by a concave one. As shown in Figure 2 (b), this behavior is well captured by our model. From Figure 2 (d), this particular envelope can be interpreted by a transition from a full lubricant film force contribution to a dominant solid force contribution. This interpretation is confirmed by measurement of the electrical contact resistance (ECR) presented in Figure 2 (c). At the beginning, the separation between solids is total (the ECR is very high) and at the end full electrical contact is observed (the ECR has a low value).

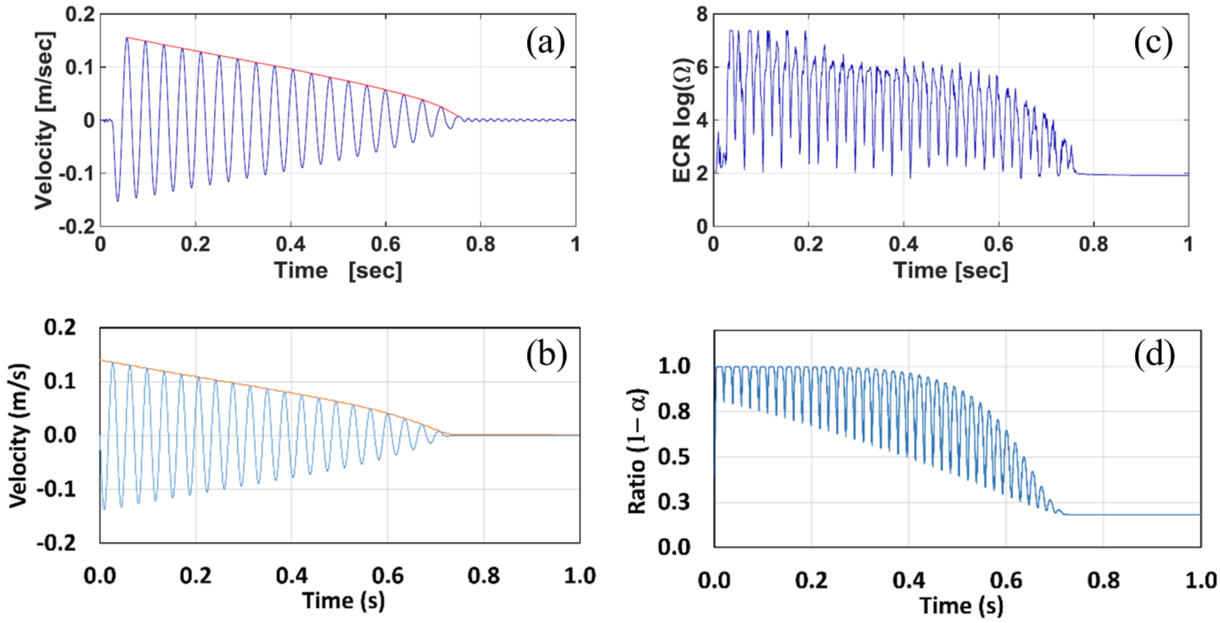

Figure 2: (a) Experimental and (b) simulated velocity responses; (c) experimental contact resistance; (d) ratio $(1 - \alpha)$ of the lubricant film force contribution.

Conclusions

The relevance of the proposed model is demonstrated in the light of comparison between numerical results and experimental ones. In particular, our new model can capture the transition related to the instantaneous film thickness which separates the two sliding surfaces, including the lag effect of the thickness dynamics. Now, ongoing work is conducted in order to relate the model parameters to the physical properties of the tribological contact.

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