Resonance Capture Cascade in Nonlinear Piezoelectric Shunt

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Summary. Nonlinear energy sinks (NESs) serve to transfer and absorb vibration energy of a mechanical system. They are locally attachted to the system and consist of a light mass, a damper and a nonlinear stiffness. Compared to linear dynamic vibration absorbers (DVA), NESs mitigate vibrations over a wider frequency band. If the mechanical system is subjected to a shock load, a NES is able to dissipate the energy of each mode separately and sequentially, from high to low frequency. This unique feature is called resonance capture cascade (RCC). When piezoelectric (PE) patches are bonded with the vibrating mechanical system, it can serve as an electrical DVA, provided a suitable shunt circuit is attached over the PE electrodes. In literature, several nonlinear shunts have been proposed and implemented. While the increased robustness has been discussed, there has been no attempt thus far to design an electrical NES specifically for resonance capture cascade. In this abstract the RCC capabilities of a mechanical NES as well as an electrical NES is shown.

Introduction

Machines are made lighter and more compact, civil structures become higher and slender. Consequently, more flexible mechanical systems are created which are susceptible to unacceptable vibration levels. To protect these mechanical systems, an auxiliary structure is attached, called a dynamic vibration absorber. It typically consists of a linear mass-spring-damper system. The linear DVA is able to mitigate a single frequency really well. However, it does so only in a narrow frequency band and as such, if the mechanical system has shifting vibration frequencies or multi-modal vibrations, the linear DVA fails to mitigate the vibrations down to an acceptable level. To increase the bandwidth of the DVA, a nonlinear energy (NES) sink has been proposed in literature [1]. The NES has a nonlinear connecting spring which typically has a cubic spring characteristic.

Resonance Capture Cascade with Mechanical NES

If a mechanical system is shock loaded, a free vibration ensues, consisting of several vibration modes. While a linear DVA is not able to decay all modes efficiently, a NES can engage in resonance capture cascade (RCC). During RCC, the NES mitigates the vibration modes sequentially, from high to low frequency [1]. More recently, the author of the current abstract [2] presented a thorough study of RCC and tuning of the NES to increase the speed of RCC. To introduce RCC, a mechanical system with 2 vibration modes is equipped with a NES, Figure 1a. The NES has the following dynamics:

$$m_{na}\ddot{x}_{na} + c_{na}(\dot{x}_{na} - \dot{x}_1) + k_{na}(x_{na} - x_1)^3 + k_{lin}(x_{na} - x_1) = 0 \tag{1}$$

with m_{na} the NES' mass, c_{na} the NES' damping, k_{na} the NES' nonlinear stiffness and k_{lin} the NES' linear stiffness. The bottom mass of the mechanical system is subjected to an initial speed of $\dot{x}_1(0) = 1$. The RCC is seen in the NES vibrations, Figure 1c, with the shifting vibration frequency best seen on its wavelet transform Figure 1d. At first, the NES vibrates with the frequency of the second mode. Then, after about 25 s, the NES shifts its frequency to the first vibration mode. After about 120 s, the NES is barely moving. The vibration in the mechanical system, Figure 1e are mitigated over the 120 s where the NES is active, with the second mode decaying first, and then the first mode. A small amount of residual energy is left after RCC.

Resonance Capture Cascade with Electrical NES as Piezo Electric Shunt

By bonding piezoelectric (PE) material to a vibrating mechanical system, the vibration energy can be transferred to electrical energy. Similar vibration mitigation performance as with mechanical DVAs can be achieved by shunting the electrodes of the PE with a suitable circuits. Because of the similar dynamical equations in mechanical and electrical domain, a NES shunt was proposed in [3]. However, no attempt thus far has been made to achieve RCC. The dynamics of a NES is highly similar to the nonlinear shunt:

$$L\ddot{q} + R\dot{q} + \frac{1}{C_3}q^3 + \frac{1}{C_1}q + V = 0 \tag{2}$$

with V the voltage over the PE electrodes, generated by the vibrations, and $q = \int I dt$ the charge. A numerical investigation now shows that RCC occurs in the electrical NES. The vibrating mechanical systems are two cantilever beams with tip masses, connected at the tip with a spring, Figure 1c. The proposed mechanical system has two dominant modes and is easy to construct experimentally. The PE patch is shunted with a NES. The electrical vibrations in the current are shown on Figure 1f and Figure 1g . The behavior is identical to the mechanical NES. The second mode is mitigated first, followed by the first mode. This can be seen as well in the tip displacement of the beam, Figure 1h.

Conclusions

Resonance capture cascade (RCC) is the unique feature of nonlinear energy sinks (NESs) that can be exploited to mitigate transient multi-modal vibrations. During RCC, vibration modes of a vibrating mechanical system are sequentially mitigated, from high to low frequency. This feature is associated with mechanical NES. In this abstract is is shown that this unique feature of the NES is also possible in shunted piezoelectric patches.

References

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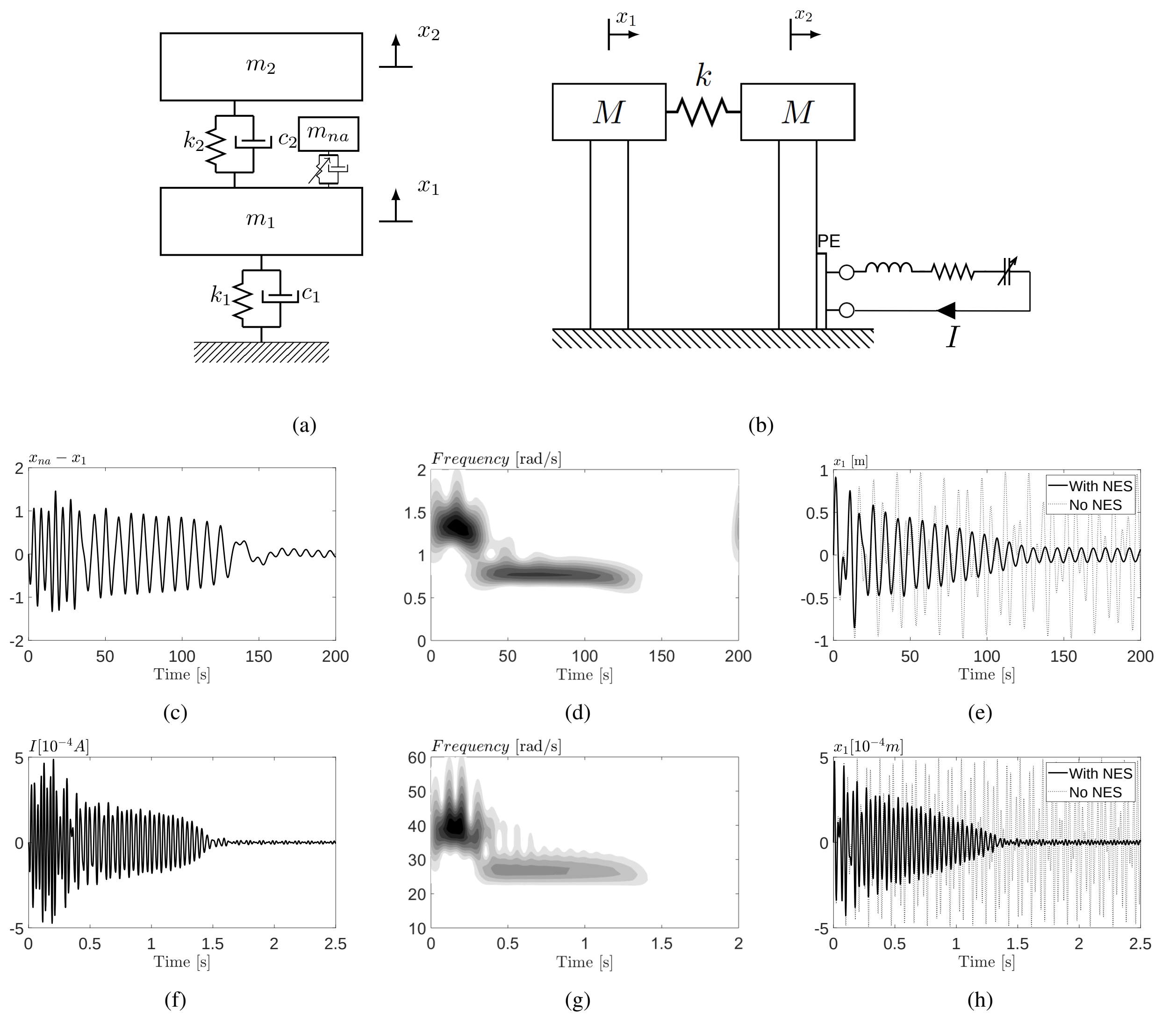

Figure 1: Mechanical system (a) with $c_1 = c_1 = k_{lin} = 0$, $m_1 = k_1 = 1$, $k_2 = m_2 = 0.2$, $m_{na} = 0.05$, $l_{na} = 0.07$ and $c_{na} = 0.02$. Mechanical system (b) consisting of two aluminum cantilever beams of 170 mm length, 35 mm width and 2 mm thick, tip masses $M = 0.033\ kg$ and connecting spring $k = 550$. The shunt consist of $L = 250\ H$, $R = 5000\ \Omega$, $C_3 = 0.25\ \mu F^3$ and $C_1 = -50\ nF$. The bonded PE patch the DURA-ACT P-876.A15 of PI [4]. The mechanical NES vibrations (c) and its wavelet transform (d). On (e), the vibrations of the first mass of (a) and on (h) the vibrations of the tip of the first cantilever beam of (b).