On the determination of high energy output operation ranges of a piezoelectric bistable energy harvesting system by parallel computing

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Summary. Nonlinearities in energy harvesting (EH) systems are introduced by intention to broaden the range of operation parameters with high energy output. Nevertheless, as multiple solutions occur due to these nonlinearities, the determination of optimal operation conditions is a challenging task. A method based on parallel computing with numerical time integration is presented to determine these optimal operational parameters in the case of a bistable piezoelectric energy harvesting system under harmonic excitation, where excitation frequency and excitation amplitude are considered. Therefore the basins of attraction are taken into account to get a measure for the occurrence of multiple solutions.

Introduction

Bistable systems have the potential for efficient EH, due to their ability to undergo so called interwell solutions with large displacements around both stable equilibrium positions and therefore high energy output in a wide range of excitation frequencies [1]. The challenge is that also intrawell solutions, i.e. solutions with small displacements around one stable equilibrium or chaotic solutions may coexist for the same parameter set. Which one of these solutions occurs is determined by the initial conditions which are in a real world EH application neither known nor controllable. To find parameter sets with likely high energy output, the probability of each solution is determined in the following. Therefore, the model of a bistable EH system first introduced by Erturk [2] in 2009 and extensively analyzed in [3] is used. It contains one mechanical degree of freedom with a cubic nonlinearity and a coupling with an electric circuit. It is given by the nondimensional equations

$$x''(\tau) + dx'(\tau) + \frac{1}{2}(-x(\tau) + x(\tau)^3) + \theta_1 U(\tau) = f\cos(\eta\tau), \tag{1}$$

$$-\frac{\Theta_2}{c_\mathrm{p}}x'(\tau) + U'(\tau) + \frac{1}{Rc_\mathrm{p}}U(\tau) = 0, \tag{2}$$

where f (normalized excitation amplitude) and η (normalized excitation frequency) are considered as the operational parameters in the following. The state variable x, depending on the normalized time τ, is proportional to the displacement of a beam, which is discretized in space by the first eigenfunction. U characterizes the voltage at the piezos. The numeric values for the dimensionless parameters are given by $d = 0.01$, $\Theta_1 = 0.13$, $\Theta_2/c_\mathrm{p} = 0.24$, $1/Rc_\mathrm{p} = 0.55$ and $R = 1$. For more detailed information see [3]. The goal is to determine harmonic excitations given by η and f, for wich the system has a high probabilty to undergo interwell solutions.

General behavior of the bistable magnetoelastic energy harvesting system

Figure 1a shows the steady state solutions and their basins of attraction of the EH system in the case $f = 0.1$ and $\eta = 0.5$. The results are computed by numerical time integration using the standard Runge-Kutta method (RK4).

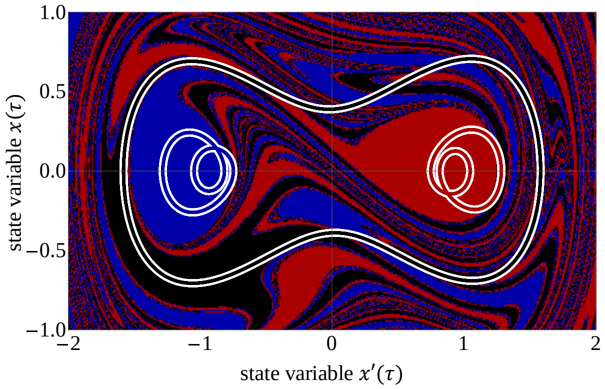

Figure 1a: Phase trajectories of intrawell solution around negative equilibrium position (blue), intrawell around positive equilibrium position (red) and interwell solution (black). The red, blue and black areas indicate initial conditions resulting in the respective solution, i.e. basins of attraction.

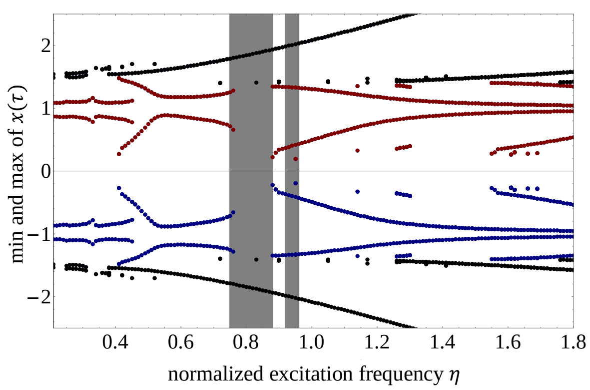

Figure 1b: Maximal and minimal values of the displacement for each stationary solution as funtion of the normalized excitation frequency η for $f = 0.1$.

The type of the solution is classified by its color, where black indicates an interwell solution and blue and red describe intrawell solutions around negative and positive equilibrium positions respectivly. In figure 1b all stationary solutions for a fixed excitation amplitude of $f = 0.1$ are shown with respect to the normalized excitation frequency η between 0.2 and 1.8. Each solution is represented by its maximum and minimum value of the state variable x. The same color code as before is applied with the addition that the gray areas indicate that there also chaotic solutions occur. It is noticeable, that for every η, where an interwell solution exists, also intrawell or chaotic solutions coexist. At this point it is so far unknown which of the coexisting solutions is most likely to occur in a real world application and therefore it is difficult to determine an operational parameter range were the energy output of the EH system is high.

Determination of high energy output operational parameter ranges

To identify operation ranges with high energy output it must be considered that each solution occurs with a different probability depending on the probability of the corresponding initial conditions, if multiple asymptotically stable solutions coexist. Therefore the coresponding basin of attraction must be taken into account. When the basin of attraction is known, it is possible to predict which solution will occur for each set of initial conditions. However, in a real world EH application the initial conditions are unknown. In this paper the probability density function $p(x_0, x_0')$ of the initial conditions is assumed as the addition of two normal distributions around both equilibrium position with a standard deviation of 0.3. If $p(x_0, x_0')$ is known, the probability P of a specific solution is

$$P = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x_0, x_0')\, b(x_0, x_0') \mathrm{d}x_0 \mathrm{d}x_0',$$

where $b(x_0, x_0')$ is an indicator function which is 1 if the specific initial conditions (x_0, x_0') are in the basin of attraction of the considered solution and 0 otherwise. To determine $b(x_0, x_0')$ using numerical integration the investigated area is limited and discretized. In our case x_0 is considered between -2.0 and 2.0 and x_0' between -1.0 and 1.0. The step size in both directions is $d = 0.008$. To compute a discretized version of the function $b(x_0, x_0')$ a numerical time integration with every possible combination of x_{0_i} and x'_{0_k} must be performed. The integration time for each integration must be long enough that the steady state solution is reached. By comparing each steady state solution with the specific one, $b\left(x_{0_i}, x'_{0_k}\right)$ can be set for all investigated initial conditions. Finally the probability of the specific solution can be determined by

$$P = \sum_{i=1}^{500} \sum_{k=1}^{250} p\left(x_{0_i}, x'_{0_k}\right) b\left(x_{0_i}, x'_{0_k}\right) d^2.$$

The results are shown in Figure 2a which is an extension of figure 1b since it additionally contains the information about the probability of each solution. The probability is visualized by the intensity of each color. For instance the interwell solution is visualized by different gray shades where white indicates a probability of 0.0 and black a probability of 1.0. It can be seen that for η in a range from 0.6 to 0.7 (green box) the probability that the system shows a high energy interwell solution is large. Hence this indicates that for a given f, η should preferably be in this area for high energy output. To achieve these results it is necessary to carry out a large number of numerical time integrations, since for each considered η (160 different values) it is required to compute the individual basin of attraction, which is generally time consuming ($500 \cdot 250 \cdot 160 = 20{,}000{,}000$ integrations). This can only be done by using a computing technique on a graphics processing unit. Therefore the time integration method (RK4) is implemented as a CUDA kernal by using the PYTHON package NUMBA. By reducing the resolution of the computed basins of attraction from $d = 0.008$ to $d = 0.08$ this approach also enables to consider further parameter variations, for example different excitation amplitudes. A corresponding result is shown in Figure 2b where the probability of the interwell solution is visualized over the normalized excitation frequency and the normalized excitation amplitude. The information in this Figure clearly demonstrates the optimal operational range of the parameters η and f for which the probability is large, that we have a high energy output.

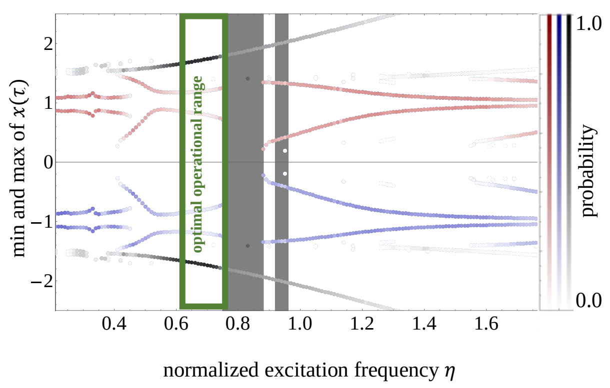

Figure 2a: Absolute maximal and absolute minimal value of x for each stationary solution with respect to η for $f = 0.1$. The opacity of each color indicates the probability of each specific solution.

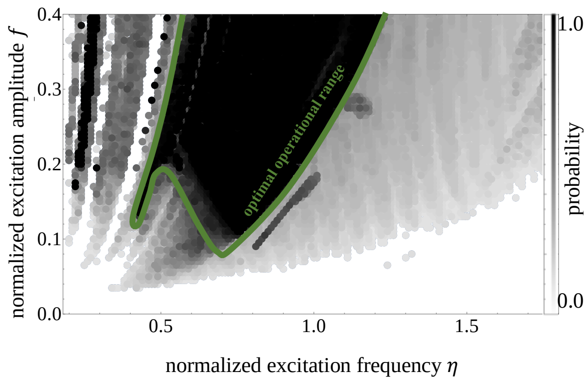

Figure 2b: Apperance probability of the interwell solution with respect to η and f. Dark black indicates a high probability for the interwell solution. Therefore dark areas indicate beneficial operation values for η and f.

References

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