Influence of mass on horizontal forced oscillations in oscillatory model of a young tree with branches

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Summary. In this paper we analysed the influence of mass on amplitudes of horizontal forced oscillations in previously developed model of complex oscillatory system that resembles corymb type of inflorescence. For describing oscillatory behavior of this system under external force influence deflection coefficients were used. The analysis was done for the case of single frequency external excitation for two different values of circular frequencies. For the value 10/s for the external force circular frequency we obtained nonlinear correlation between amplitudes and masses.

Introduction

Plants and trees oscillate under the influence of wind. This is particularly important for young seedlings that can be damaged before the plant is fully rooted in the ground. Oscillations of tree stem of living trees can be studied through different approaches [1, 2], where it is possible to measure the natural frequencies of trees [3]. Type and structure of wood structure (material properties of the system that could be considered as complex oscillatory system) and characteristics of wind (an external force) may contribute to the damage of a tree and cause it to crack or fall. Mechanical stability of trees is thus a very important problem [4]. Young tree seedlings are of particular interest in urban area exposed to the wind. Recently we developed the model of a complex oscillatory systems that was inspired by corymb type of inflorescence [5]. We use this model (Fig1) to study the influence of mass of tree branches on amplitudes of horizontal forced oscillations at the certain points of the tree stem.

Description of the model

The system could be assumed as ideally elastic system consisting of material particles on rigid massless rods with length ℓ_i with the angle β_i to light elastic console with length ℓ. Length of the console corresponds to the tree stem, and length of the massless rods to the tree branches. Material particles at the end of the massless rods correspond to the mass of the branch. The approximations of this system are: connections between rigid massless rods and elastic console are rigid, oscillations of console and system are small, tilts of tangent to elastic axis of banded console are very small and negligible, tilts of console cross-section during console banding according to x and y axis of cross section are very small and negligible. Masses of material particles are equal. System oscillates in horizontal and vertical plane. Bending stiffness of the elastic console is equal in horizontal and vertical plane. For describing oscillatory behavior of this system influence coefficients of deflection were used. Displacement influence coefficients were determined on the basis of equations of elastic line of console load with unit force and unit momentum. Forced oscillations of the system can be described by two independent subsystems of ordinary differential equations in vertical and horizontal plane. Each subsystem of ordinary differential equations consists of four coupled ordinary differential equations of second order in the following form:

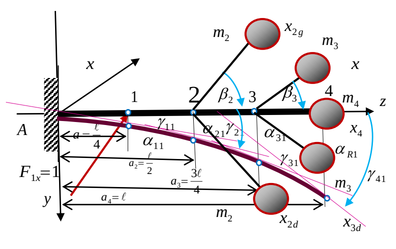


Figure 1. Complex oscillatory systems that can be used for studding oscillations of tree with branches. (Taken from ref [1].

Table 1. Resonant values of oscillations of the console for different masses of single material points (branches)

Resonant values Ωy(1/s)	1kg	1.3kg	1.6kg	1.9kg	2.2kg	2.5kg	2.8kg
R1	5.924	5.196	4.724	4.325	4.028	3.771	3.556
R2	3.435	3.013	2.681	2.427	2.301	2.158	1.559
R3	8.074	7.049	6.383	5.837	5.425	5.113	4.837

For oscillations in horizontal plane:

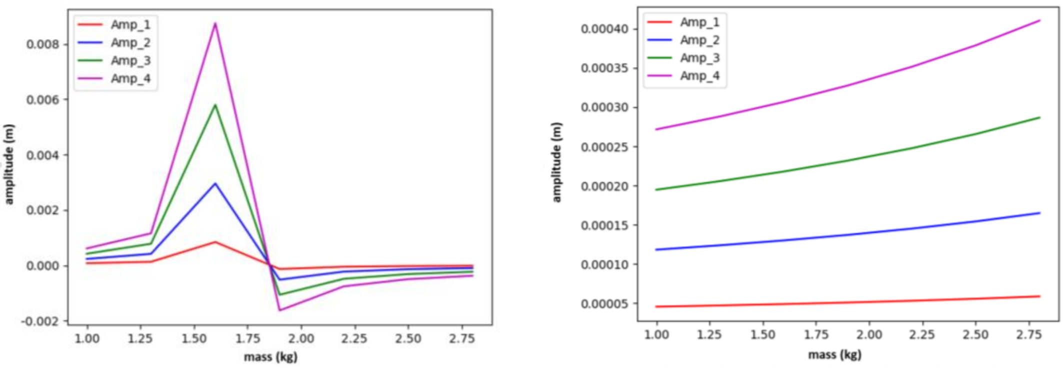
$$x_i = \alpha_{i1} F_{01x} \sin \Omega_x t + \alpha_{i2}\left(-2m_2\ddot{x}_2\right) + \alpha_{i3}\left(-2m_3\ddot{x}_3\right) + \alpha_{i4}\left(-m_4\ddot{x}_4\right) + \\ + \delta_{i2}\left(-2m_2\ddot{x}_2\ell_2 \sin\beta_2\right) + \delta_{i3}\left(-2m_3\ddot{x}_3\ell_3 \sin\beta_3\right) \qquad i = 1 \ldots 4 \qquad (1)$$

Where α_{ik} and δ_{ik} are influence coefficients of deflection on cross section "i" under unit force $F_k = 1$ and under unit momentum. $M_{k}=1$ respectively, on section 'k'. For calculating the influence coefficients of deflection the bending stiffness of the elastic console is needed and is equal to: $\mathrm{B}_x=\mathrm{B}_y=EI_x=EI_y=E\frac{r^4\pi}{4}$ where $I_x=I_y=\frac{r^4\pi}{4}$ is the axial moment of inertia of the surface of the cross-sectional area for the corresponding central axis passing through the center of the circle. E is Young's module of elasticity, r is half a diameter of the console.

We analyze influence of mass of tree branches on amplitudes of horizontal forced oscillations at the certain points of the tree stem. Numerical analysis was done for the following data: E=10,7x10^{9}Pa, r=0,015m, $\ell_2=\ell_3=0.55m$, $\ell=1.5m$, $\beta_2=\pi/4$ $\beta_3=\pi/6$; $\Omega_y=5/s$ and for $\Omega_y=10/s$, for different masses (from 1-3kg with step 0.3).

Results

Dependences of amplitudes of masses of material particles on their ends (branches) for two values of circular frequency of external force are shown on Fig.2. Graphs were obtained by calculating all the values numerically using Python programming language. Library numpy was used for determinant calculation, while matplotlib was used for graph plotting. In the Table 1. tree resonant values are shown for each mass value for which characteristic local minimum and maximum of amplitudes in certain point of the complex oscillatory model are obtained when circular frequency of the external force is $\Omega_y=10/s$.



A B

Figure 2. A. Amplitudes of oscillations of four different points for different masses of material particles for $\Omega_y=10/s$.
B. Amplitudes of oscillations of four different points for different masses of material particles for $\Omega_y=5/s$.

Conclusions

We analysed influence of mass on amplitudes of horizontal oscillations of a complex oscillatory model of young three with branches under external force with circular frequency of $\Omega_y=5/s$ and $\Omega_y=10/s$. When circular frequency of the external force is 5/s, which is around resonant frequencies for most of the selected masses we obtained constantly increasing amplitudes of forced horizontal oscillations of our complex oscillator. For the case when circular frequency of the external force is 10/s, amplitudes of forced horizontal oscillations of our complex oscillator have nonlinear character showing local maximum and minimums. Maximum amplitude for the chosen data systems is for mass 1.6 kg per branch. Absolute minimum of the amplitude is for mass 2.8 kg per branch. Specific combination of the parameters of the system and circular frequency of the external force determine the maximum amplitude of forced oscillations and it has a nonlinear character.

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