Nonlinear Analysis of the Snaking Motion of Towed Vehicles

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Summary. A mechanical model is introduced in order to investigate the snaking/rocking motion of towed two-wheeled trailers. The sense of the Hopf bifurcations related to the linear stability boundaries of the rectilinear motion are investigated numerically. The center manifold reduction is also performed semi-analytically, by which the relevance of the pitch motion is identified.

Introduction

The instability of towed vehicles is a relevant safety risk on the roads. In case of a badly chosen towing velocity and/or a badly loaded trailer, the vehicle may start a so-called snaking motion [1, 2]. The trailer may even start rocking on its wheels, and the rollover of trailer can happen.
In-plane mechanical models of towed vehicles are deeply analyzed in the literature (see, for example, [3]) and it is established that the Hopf bifurcation is subcritical at the linear stability boundary located at small towing velocities. Here, the trailer is modeled with a spatial, 4 degrees-of-freedom (DoF) mechanical model given in [4], namely, the pitch and roll motions are also considered when the nonlinear vibrations (e.g. the rocking motion) are analyzed semi-analytically and numerically.

Mechanical Model

The mechanical model of the two-wheeled trailer can be seen in Fig. 1. The trailer is towed with constant velocity v in X direction at the king pin. The motion of the trailer is described with the yaw angle ψ, the pitch angle ϑ, the roll angle φ and the lateral displacement of the king pin u.

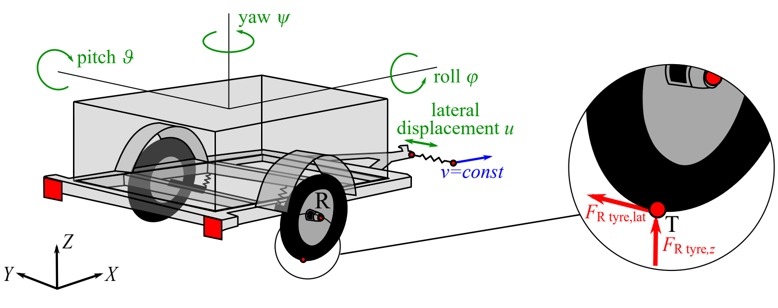

Figure 1: The mechanical model of the trailer with the generalized coordinates and the tyre forces.

Since there are only geometric constraints, the system is holonomic. Thus, the equations of motion can be derived from the Lagrange equation of the second kind (for details see [4]). The lateral tyre forces are considered as

$$F_{\text{tyre,lat}} = \mu(\alpha) F_{\text{tyre},z} \,, \tag{1}$$

where $F_{\text{tyre},z}$ is the vertical load on the tyre and $\mu(\alpha)$ involves Pacejka's Magic Formula [6]. The side slip angle α is calculated by means of the longitudinal and lateral components of the velocity of the contact point T of the wheel:

$$\alpha = -\arctan\left(\frac{v_{\text{T,lat}}}{v_{\text{T,long}}}\right) . \tag{2}$$

Linear Stability Analysis and Bifurcation Analysis

Based on the linear stability analysis of the rectilinear motion, it can be concluded that the linearised system can be separated into two subsystems: the pitch motion can be decoupled as a 1 DoF subsystem, while remaining equations form a 3 DoF subsystem. Thus, the pitch motion does not affect the linear stability of the rectilinear motion. But, it has effect on the nonlinear vibrations.
Asymmetry is introduced into the system by the lateral tyre force characteristics formulated in (1). The vertical load $F_{\text{tyre},z}$ on the tyres depends linearly on the pitch angle ϑ, as it can be seen in the left panel of Fig. 2. The coefficient $\mu(\alpha)$ is an even function (see the right panel of Fig. 2), thus, the lateral tyre force contains mixed second degree terms with respect to the generalized coordinates and velocities. When the bifurcation analysis is carried out with center manifold (CM) reduction, it can be identified that the pitch motion influences the sense of the Hopf bifurcation through these second degree terms.

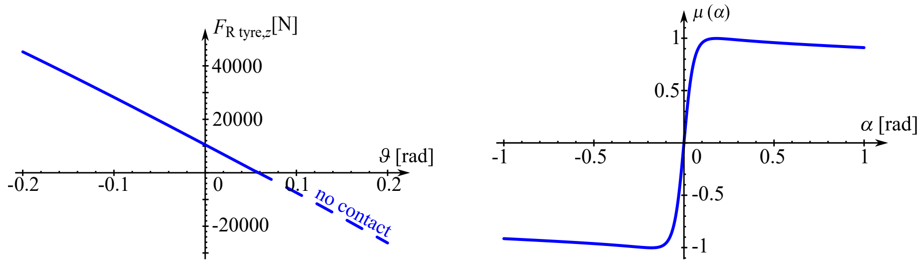

Figure 2: The characteristics of the vertical tyre forces and characteristics of the lateral force defined by the Magic Formula in $\mu(\alpha)$ [6].

The stability of the periodic orbits were also investigated by continuation in *DDE Biftool* [5]. It was confirmed that supercritical Hopf bifurcation exists at small velocities, while in-plane trailer models of the literature provides a subcritical one. To check our hypothesis on the effect of the pitch motion, a reduced model was also analyzed in which the pitch motion was blocked ($\vartheta \equiv 0$).

The nature of the periodic solutions, namely, the sense of the Hopf bifurcations can be seen in Fig. 3 for the spatial, 4 DoF model and for the reduced model. In the stability charts, the blue continuous and red dashed lines correspond to supercritical and subcritical bifurcations, respectively. In the figures, parameter f describes the vertical position of the center of mass. As it can be observed in the left panel, the periodic solution is stable at smaller critical speed in case of the 4 DoF model, thus, the bifurcation is supercritical. On the contrary, in the right panel, the periodic solution is unstable (subcritical Hopf bifurcation occurs) for the same critical speed when the reduced model is considered.

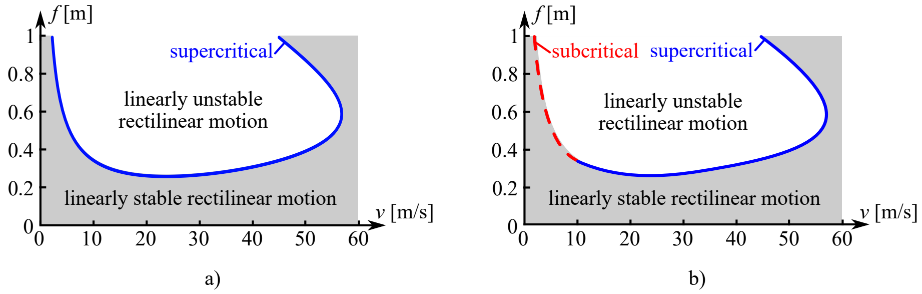

Figure 3: The stability of the periodic solutions a) for the spatial, b) for the reduced model.

Conclusions

Based on the semi-analytical bifurcation analysis, the effect of the pitch motion on the stability of the emerging periodic solutions is verified. It is shown that the in-plane trailer models of the literature can provide different results with respect to the sense of the Hopf bifurcation at small velocities.

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