Harmonic balance-based crack size estimation in an ultrasonic fatigue specimen

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Summary. Closing-opening cracks during ultrasonic fatigue tests are suspected to generate nonlinearities in the vibration signal. Due to the periodicity of the ultrasonic transducer and choice of non-smooth stiffness nonlinearity, the Harmonic Balance method is utilized to simulate the resulting 1DOF dynamic model. The energy that transferred from the fundamental harmonic to higher harmonics is represented as a harmonic amplitude ratio parameter and is utilized to inverse model a nonlinear crack parameter for an experimental signal. A feature of this dynamic model allows for the addition of nonlinear effects that evolve with the fatigue specimen's life.

Introduction

During ultrasonic fatigue tests, the evolving spectrum of a vibration signal is suspected to contain essential information about the nature of fatigue damage, such as crack initiation time and actual size. Traditionally, the properties studied from this information are the second harmonic, the eigenmodes, or the extremely small variations of resonance frequency. These vibration properties are affected by micro-plasticity and by the formation of cracks which can therefore be used to detect micro-structural changes [1, 2].
The approach proposed here consists of modeling the dynamical response of the standing wave of the system. A key feature of a cyclically closing-opening crack is an instantaneous, nonlinear, and very local change of stiffness of the system. This nonlinearity leads to a nonlinear dynamic response of the system and, as a consequence, manifestation of higher-harmonic generation in the vibration response. A closing-opening crack's restoration force is typically modeled by an asymmetric piecewise linear or bilinear stiffness [3]. Thus the goal of this study is to inverse model a nonlinear crack parameter γ using the Harmonic Balance method (HBM) [4] to obtain a nonlinear dynamic response of the system. Ideally, this can allow for real-time crack initiation and possibly size evolution monitoring.

Harmonic Balance dynamic modeling

During ultrasonic fatigue testing, seen in fig. 1(a), repeatedly transmitted ultrasonic waves at a resonant or anti-resonant frequency form a longitudinal standing wave in the fatigue specimen [1]. The undamaged fatigue specimen can be schematized as an equivalent 2 degree-of-freedom (DOF) lumped mass-spring oscillator. The oscillator's total mass $2m$ and stiffness k are tuned to resonate at the specimen's first longitudinal mode. The phase difference between the excitation and vibration is minimized by a phase-locked loop (PLL) within the converter. A longitudinal standing wave is produced with peaks at specimen's extremities, i.e., the oscillator's masses. This phenomenon has two consequences: First, the power required from the converter can be minimized. Second, the specimen's motion allows for a half specimen representation, or 1DOF system, that is dynamically modeled with respect to the specimen's elongation, i.e., a mass-spring excited by base motion at an anti-resonance frequency $\omega_{\text{anti}} = \sqrt{2}\omega_{\text{res}}$ with $\omega_{\text{res}} = \omega_0\sqrt{1-\zeta^2}$ [5], where $\omega_0 = \sqrt{k/m}$.

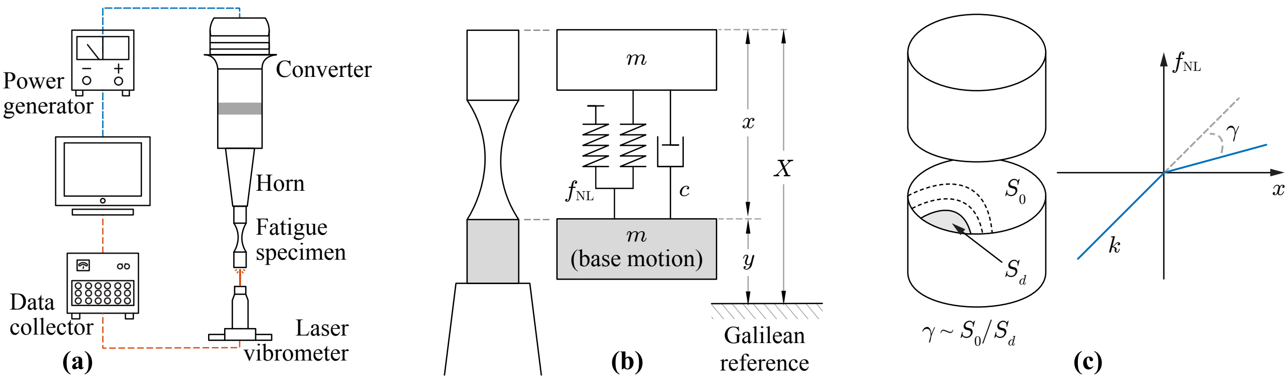

Figure 1: (a) Schematic of ultrasonic fatigue test experimental setup, (b) fatigue specimen model with absolute displacement of mass X and Galilean reference displacement for mass x and base y, and (c) concept of crack nonlinearity parameter $\gamma \sim S_d/S_0$ with plot of the bilinear stiffness restoration force f_{NL}

The 1DOF system has the equation of motion:

$$m\ddot{X} = F, \quad \text{with} \quad F = -c\dot{x} - f_{\text{NL}} = -c\dot{x} - k(1-\gamma H)x, \tag{1}$$

where m, X, F, represents the upper mass, the absolute displacement of the system, and the force transmitted to the upper mass through the central part of the specimen respectively. The fatigue specimen is forced at the base, with displacement y, seen in fig. 1(b). The force F is obtained by splitting X into the relative displacement of the fatigue specimen free-end x and base (lower mass) motion y. When the exerted force on the mass is due to base excitation, eq. (1) becomes:

$$m\ddot{x} + c\dot{x} + k(1-\gamma H)x, = -m\ddot{y}, \tag{2}$$

with γ suspected to be correlated to the crack area ratio (see fig. 1(c)) and H represents the Heaviside function evaluated for values of x. It is assumed that the base moves harmonically with the amplitude of the base's motion U, and the frequency of the base's motion, ω_b, such that $y(t) = U\cos(\omega_b t)$. Eq. (2) is solved via HBM, yielding a finite Fourier decomposition for x, with j harmonics:

$$x \approx \mathrm{Re}\left(\sum A_j e^{ij\omega_{\text{anti}}t}\right). \tag{3}$$

Since the standing wave is distorted as it passes through the crack nonlinearity, energy is transferred from the fundamental harmonic to the higher harmonics. Thus, a harmonic amplitude ratio α is defined at the anti-resonant frequency ω_{anti} as:

$$\alpha_n = \frac{A_n(\omega_{\text{anti}})}{A_1(\omega_{\text{anti}})}, \quad \text{for } n > 1, \tag{4}$$

where peak amplitudes A_1 and A_n correspond to the fundamental and n^{th} higher harmonics, respectively. This anti-resonant frequency is assumed to be the frequency of the base's motion.

Experimental details and results

A short-time Fourier transform (STFT) algorithm [6] is utilized to extract the first five harmonics and the anti-resonant frequency from the down-sampled velocity vibration signal, seen in fig. 2(a). Consequently, $\alpha_2', \ldots, \alpha_5'$ is calculated using the relation in eq. (4) with $(\,\cdot\,)'$ representing an experimentally determined parameter. The dynamic model, seen in eq. (2) , is computed via HBM for $1\text{e}{-4} < \gamma < 2\text{e}{-1}$ and the corresponding $\alpha_2, \ldots, \alpha_5$ are extracted. The dynamic model's γ, α relationship are curve fitted with first-order power equations. Experimental $\alpha_2', \ldots, \alpha_5'$ are found with their respective curve fits, seen in fig. 2(b). Under modeling assumptions, γ' at the zero$^{\text{th}}$ cycle is undamaged. Thus, fig. 2(c) shows the evolution of damage parameters $\gamma_2', \ldots, \gamma_5'$. The second and third harmonics are seen to follow similar paths until 8e5 cycles in which they diverge exponentially. At cycles nearing fatigue, the third, fourth, and fifth harmonics rapidly grow, with the fifth harmonic the quickest. Despite γ' not equal throughout the evolution, the order of magnitude is consistent between the different harmonics. The authors estimate that there is a magnitude factor of approximately 1e2 that would lead to a measurement of crack size evolution if $\gamma S_0 = S_d$.

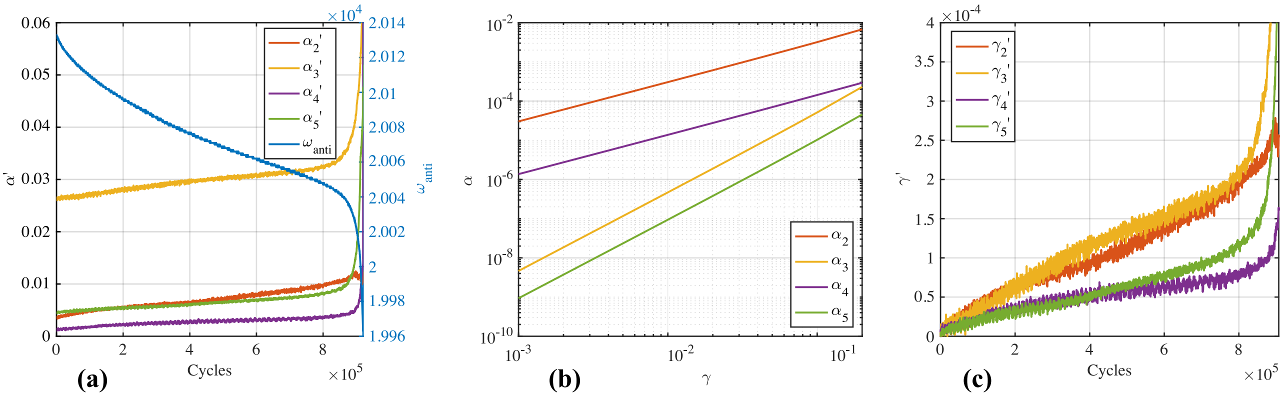

Figure 2: (a) Experimental α' (left axis) and experimental ω_{anti} (right axis) versus cycles, (b) HBM simulated γ versus α first-order power equations fits, and (c) experimental γ' versus cycles

Conclusions

The nonlinear crack parameter γ was calibrated with a bilinear stiffness dynamic model. α is computed with HBM for multiple harmonics and compared with experimental γ's versus cycles. Bilinear stiffness can qualitatively describe nonlinear behaviors for the second and third harmonics for cycles before approximately 8e5, but other nonlinearities are influencing higher harmonics and at cycles nearing failure. The form of eq. (3) allows for additional non-linearities to dynamically model the fatigue behavior of higher harmonics in conjunction with bilinear stiffness.

References

[1] Kumar A., Adharapurapu R.R., Jones J.W., Pollock T.M. (2011) In situ damage assessment in a cast magnesium alloy during very high cycle fatigue. *Scripta Mater* **64**:65-68.

[2] Heinz S., Balle F., Wagner G., Eifler D. (2013) Analysis of fatigue properties and failure mechanisms of Ti6Al4V in the very high cycle fatigue regime using ultrasonic technology and 3D laser scanning vibrometry. *Ultrasonics* **53**:1433-1440.

[3] Broda D., Staszewski W.J., Martowicz A., Uhl T., Silberschmidt V.V. (2014) Modelling of nonlinear crack-wave interactions for damage detection based on ultrasound – A review. *J Sound Vib* **333**:1097-1118.

[4] Sarrouy E., Sinou J.-J. (2011) Non-linear periodic and quasi-periodic vibrations in mechanical systems – on the use of the harmonic balance methods. In: Advances in Vibration Analysis Research. InTech.

[5] Inman D.J. (2014) Base excitation, in: Engineering Vibration. Pearson.

[6] Allen, J.B., Rabiner, L.R., (1977) A unified approach to short-time Fourier analysis and synthesis. *P IEEE* **65**:1558-1564.