# Stabilizing reset control for motion systems with Stribeck friction

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*Summary*. We present a reset control approach to achieve setpoint regulation of a PID-based motion system subject to Coulomb and velocity-dependent friction, including the velocity-weakening (Stribeck) effect. While classical PID control results in persistent oscillations around the setpoint (hunting), the proposed reset mechanism ensures asymptotic stability. Robustness for unknown static friction levels, and an unknown Stribeck contribution is obtained. The working principle of the controller is demonstrated experimentally on a motion stage of an electron microscope, showing superior performance over classical PID control.

## Introduction

We present a reset integral control approach for stabilization of motion systems with unknown Coulomb and Stribeck friction. Friction is a performance-limiting factor in many high-precision motion systems, in the sense that it limits the achievable positioning accuracy and the settling times. Many different control techniques for frictional motion systems have been presented in the literature. Several control solutions rely on developing as-accurate-as-possible friction models, used for online compensation in a control loop, see, e.g., [1]. Also non-model-based solutions have been proposed, e.g., impulsive control (see [2]) or sliding-mode control.

Despite the availability of a wide range of control techniques for frictional systems, linear controllers are still used in the vast majority of industrial motion systems due to the existence of intuitive design and tuning tools. In the industry, the classical proportional-integral-derivative (PID) controller is most commonly used for motion systems with friction. In particular, the integrator action is capable of compensating the unknown static friction, due to the the control force increment arising from integrating the position error. However, PID control does not generally achieve stability in the presence of Stribeck friction, resulting in poor positioning accuracy. While the integrator action compensates for the static part of the friction, friction overcompensation occurs as the velocity increases due to the velocity-weakening effect. As a result, the system overshoots the setpoint and ends up in stick-slip oscillations (*hunting*). In order to eliminate these persistent oscillations, we propose a reset integral controller that induces asymptotic stability of the setpoint, despite the presence of *unknown* static friction, and an *unknown* velocity-weakening effect in the friction characteristic.

## Reset controller design

Consider a single-degree-of-freedom mass $m$ sliding on a horizontal plane with position $z_1$ and velocity $z_2$, subject to a control (force) input $u$ and a friction force belonging to a set $\Psi(z_2)$, governed by the dynamics

$$\dot{z}_1 = z_2, \quad \dot{z}_2 \in \frac{1}{m}\left(\Psi(z_2) + u\right), \quad z_2 \rightrightarrows \Psi(z_2) := -F_s \operatorname{Sign}(z_2) - \alpha z_2 + f(z_2), \tag{1}$$

where $F_s$ is the static friction, $\operatorname{Sign}(\cdot)$ is the set-valued sign function (i.e., with $\operatorname{Sign}(0) := [-1,1]$), $\alpha z_2$ the viscous friction contribution (where $\alpha \geq 0$ is the viscous friction coefficient), and $f$ is a nonlinear velocity-dependent friction contribution, encompassing the Stribeck effect. Define the setpoint $(z_1, z_2) = (r, 0)$ for any constant position reference $r$. Let us first present a *classical* PID controller for input $u$ in (1), i.e.,

$$u = -k_p(z_1 - r) - k_d z_2 - k_i z_3, \quad \dot{z}_3 = z_1 - r, \tag{2}$$

where $z_3$ is the integral state of the PID controller, and $k_p$, $k_d$, $k_i$ represent the proportional, derivative and integral gains, respectively, satisfying $k_p > 0$, $k_i > 0$, and $k_p k_d > m k_i$. Finally, we embrace the (mild) assumption that the friction characteristic satisfies $|f(z_2)| \leq F_s$ for all $z_2$, that $z_2 f(z_2) \geq 0$ for all $z_2$, that $f$ is globally Lipschitz with Lipschitz constant $L > 0$, and that, for some (potentially arbitrarily small) $\varepsilon_v > 0$ and $L_2 \in (k_d + \alpha, L]$, $f(z_2) = L_2 z_2$ for all $|z_2| \leq \varepsilon_v$.

In order to achieve closed-loop stability, we enhance the integrator in (2) with *resets*. The integrator performs two particular resets, where the key mechanism of these resets is to enforce that the integrator control force (given by $k_i z_3$) *always points in the direction of the setpoint*. To this end, we introduce a boolean state $b \in \{-1, 1\}$, characterizing whether the mass moves *towards* the setpoint (then $b = 1$), or *away from* the setpoint (then $b = -1$, typically after an overshoot of the position error). Then, the inequality $b z_2(z_1 - r) \leq 0$ is always satisfied. The first reset that we propose entails a sign change of the integrator state $z_3$ at a zero-crossing of the position error $z_1 - r$. We also toggle $b$ at this instant, because a zero-crossing of the position error marks the start of an overshoot phase, i.e.,

$$z_3^+ = -z_3, \quad b^+ = -b, \quad \text{when } z_1 - r = 0 \text{ and } b = 1, \tag{3}$$

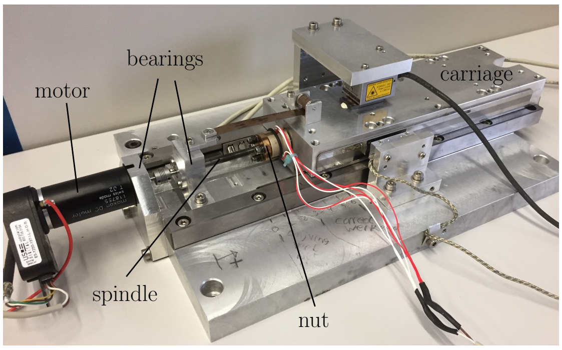


Figure 1: Experimental setup.

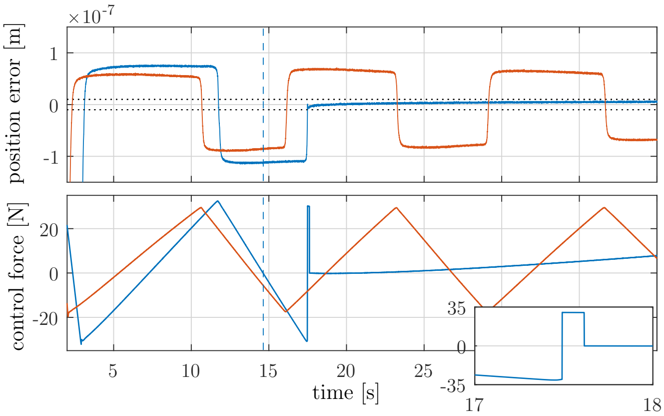


Figure 2: Position error response and experimental control force with the classical PID (red), and the reset PID (blue).

where the notation "$x^+$" represents the value of the considered state $x$ after an instantaneous change, i.e., the controller reset. Besides recovering stability of the setpoint, the reset in (3) results also in some overshoot reduction. The second reset involves resetting to zero the integrator state $z_3$ when the velocity $z_2$ hits zero *after the overshoot*, i.e.,

$$z_3^+ = 0, \quad b^+ = -b, \quad \text{when } z_2 = 0 \text{ and } b = -1. \tag{4}$$

The reset in (4) also results in a reduction of the duration of the stick phases occurring when the mass stops after the overshoot. Summarizing, the closed-loop system with the proposed reset PID controller is given by (1)-(4).
Closed-loop stability is analyzed as follows. First, we write the closed-loop system in the hybrid systems framework of [3], where we use a *hybrid* description of the Coulomb friction element in (1), as presented in [4]. We then prove that solutions to the closed-loop system (1)-(4) are also contained in said hybrid model. We then exploit the hybrid model to show that (under the aforementioned mild assumptions on the controller gains and friction characteristics) the setpoint is globally asymptotically stable, using a Lyapunov function and a recent hybrid invariance principle [5].

## Experimental case study

We demonstrate the working principle and the effectiveness of the proposed reset controller on an industrial motion platform (a sample manipulation stage of an electron microscope), see Figure 1. A servo motor is connected via a spindle and a nut to a carriage, whose position is measured by a linear encoder. The goal is to position the carriage within a desired accuracy band of 10 nm. The main sources of friction are two bearings supporting the motor axis, and the contact between the spindle and the nut. We have implemented both the classical PID controller, and the proposed reset PID controller (with the same gains). For the latter one, we have designed suitable robustified conditions capable of triggering the controller resets (3), (4) also in the presence of measurement noise. Figure 2 shows the position error response and the control force for an experiment with the *classical* PID controller (red), and with the proposed *reset* PID controller (blue). The classical PID controller induces a persistent oscillation, limiting the achievable setpoint accuracy. For the reset controller experiment, the reset enhancements are activated at the time instant indicated by the blue vertical dashed line (up to this time instant, a classical PID controller is active). Then, the challenging desired accuracy band of 10 nm, indicated by the horizontal black dotted lines in the top subplot, is achieved after two controller resets. The corresponding control force is discontinuous due to the controller resets, visualized in the lower subplot and highlighted in the inset.

## Conclusions

We proposed a reset PID control strategy for motion systems with unknown Coulomb and velocity-dependent friction, including the Stribeck effect. Our reset strategy recovers global asymptotic stability of the setpoint (lost by the hunting effect associated to the classical PID). The working principle and effectiveness of the controller are experimentally demonstrated on an industrial high-precision positioning device.

## References


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