Simulation of Road Surfaces Profiles by a Stochastic Parametrical Model

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Summary. Road irregularities have an important influence on the dynamic behavior of vehicles. Knowledge of their characteristics and magnitude is essential for the design of the vehicle. The problem of interest is the simulation of road surfaces profiles because modern test facilities and computer simulations of vehicle dynamics needs driving excitations. An import issue is the power spectral densities and the approximation by analytical formulas. In the paper a stochastic parametrical nonlinear model of first order with bounded amplitudes will be discussed. Some analytical and numerical results will be shown.

Road Surface Profiles

The road surface profiles are defined by ISO8608 Mechanical vibration – Road surface profiles – Reporting of measured data [1]. Figure 1 shows an example of a measured road.

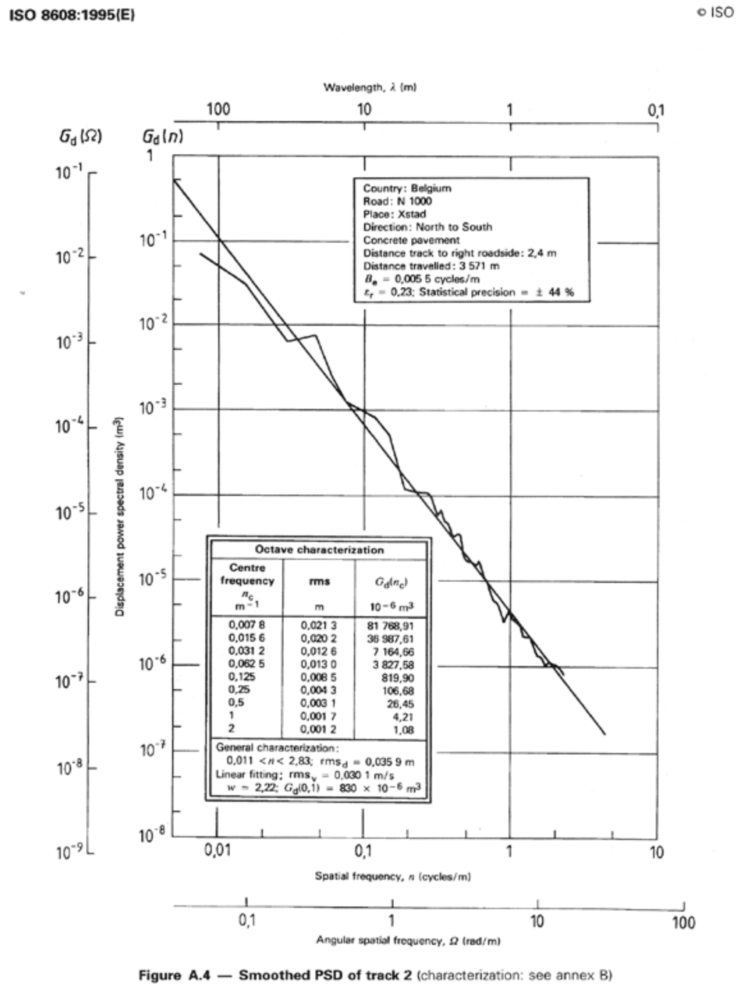

Figure 1: Measured road surface profile [1]

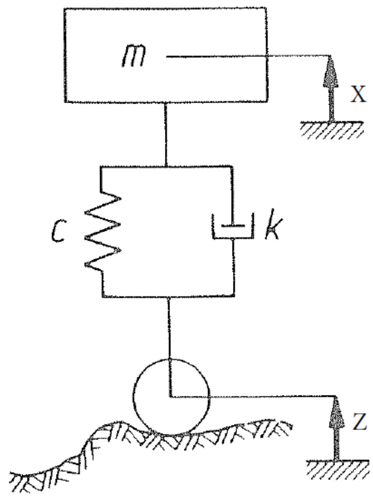

Figure 2: A quarter car (1 DOF) with road surface profile Z_t

Stochastic Parametrical Model

The stochastic parametrical nonlinear model of first order

$$\dot{\mathrm{Z}}_{\mathrm{t}} = \left(\frac{1}{2}\sigma^2 - \omega_0\right) Z_t + \sqrt{Z_0^2 - Z_t^2}\,\sigma\dot{\mathrm{W}}_{\mathrm{t}}$$

with σ intensity of white noise, Z_0 the maximum amplitude, ω_0 the corner frequency and W_t the Wiener process with the mean value $\mathrm{E}\{dW_t\} = 0$ and the variance $E\{dW_t^2\} = dt$. The stochastic differential equation (Ito) is

$$\mathrm{d}Z_t = -\omega_0 Z_t \mathrm{dt} + \sqrt{Z_0^2 - Z_t^2}\, \sigma dW_{\mathrm{t}} \qquad |Z_t| \leq Z_0$$

The Fokker Planck equation of the probability density function is

$$\frac{\partial \mathrm{p}}{\partial \mathrm{t}} - \omega_0 \frac{\partial}{\partial z}[zp] - \frac{\sigma^2}{2}\frac{\partial^2}{\partial z^2}[(Z_0^2 - z^2)p] = 0$$

The stationary solution of the density function can be calculated to

$$\mathrm{p(z)} = C\,(Z_0^2 - z^2)^{(-1+\omega_0/\sigma^2)} \qquad \sigma^2 < \omega_0$$

The constant C fulfilled the normalization condition $\int_{-Z_0}^{+Z_0} p(z)dz = 1$. The auto correlation function of the stationary process Z_t is

$$R_z(\tau) = \sigma_z^2\, e^{-\omega_0|\tau|}$$

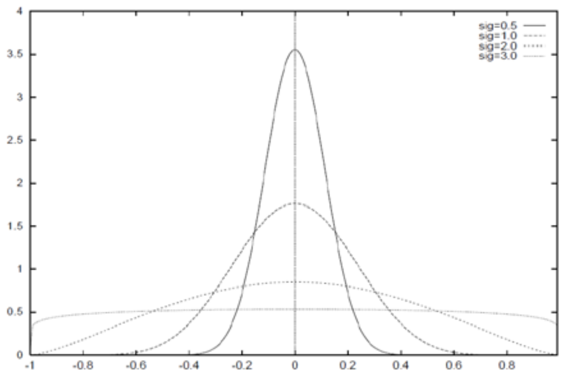

Figure 3: Stationary density distributions p(z) with $\omega_0 = 10$, $Z_0 = 1$ and different σ

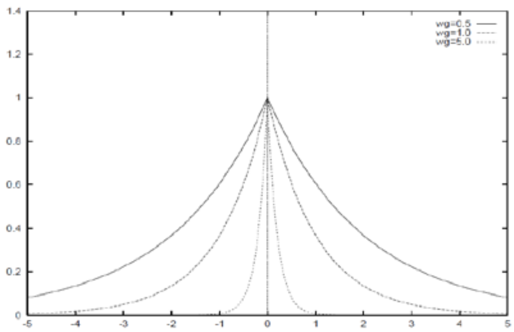

Figure 4: Auto correlation function $R_z(\tau)$ with $\sigma_z = 1$ and different ω_0

A numerical realization of the stationary process Z_t with the parameters $\omega_0 = 10$, $Z_0 = 1, \sigma = 0.5$ and $N = 10^6$ time steps shows figure 3.

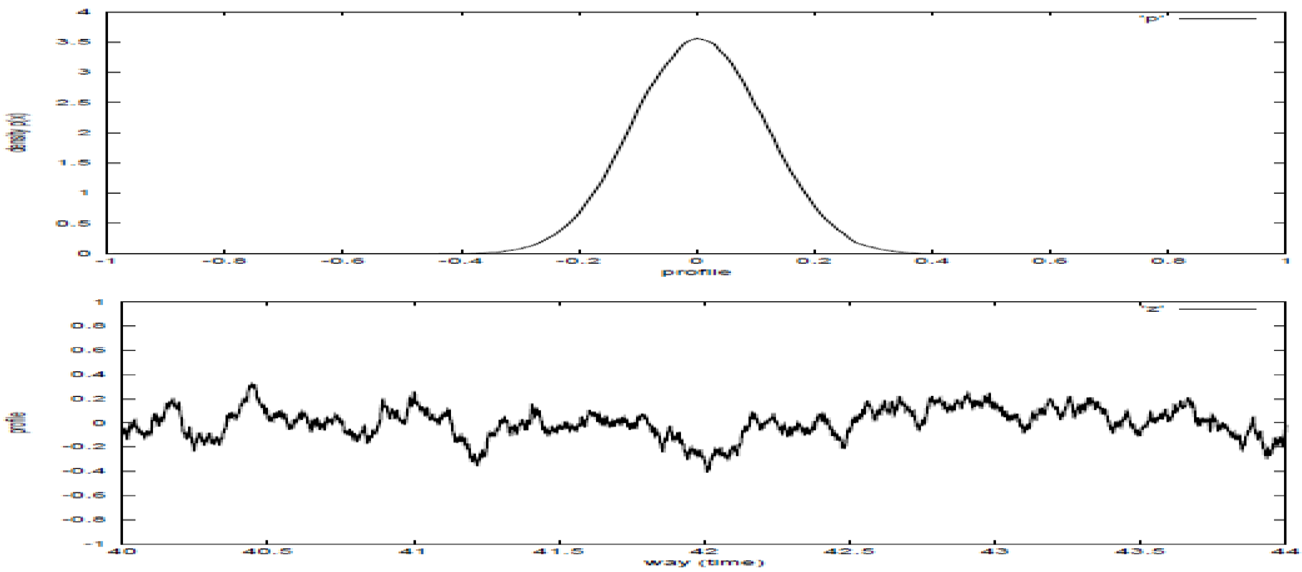

Figure 3: Numerical realization of the stationary process Z_t ($\omega_0 = 10$, $Z_0 = 1, \sigma = 0.5, N = 10^6$)

References

[1] ISO 8608:1995(E), Mechanical vibration - Road surface profiles - Reporting of measured data.
[2] Wedig W.: Dynamics of Cars Driving on Stochastic Roads, In: Spanos P., Deodatis G. (Eds), CSM-4., Rotterdam, Millpress, p. 647 - 654, 2003.
[3] Doods C. J., Robson J. D.: The Description of Road Surface Roughness, Journal Sound and Vibration, 31, p. 175 - 183, 1973.
[4] Arnold L.: *Stochastic Differential Equations*. New York, Wiley, 1974