Chaos in a non-linear non-buckled microresonator

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Summary. Chaos is a phenomenon describing the complex dynamics of many systems, from the evolution of the weather to the dynamics of cosmic entities. Since a few decades, generating chaos from a physical system has triggered a lot of research, especially in the optical domain. In the mechanical domain, chaos generation has been investigated mainly with buckled structures. These bistable systems enter in a chaotic regime upon the application of a strong enough alternative force. However, in the micromechanical domain, buckling a structure is demanding and typically requires a large voltage, incompatible with available technology. In this paper, we describe a new way of generating chaos from a Micro Electro-Mechanical System (MEMS) using the dynamical bistability of a nonlinear system, activated by a modulated signal within the resonance of the system. We measured the generated chaos experimentally with a microresonator, and characterized it with Poincaré sections and Lyapunov exponent measurements. In our case, the chaos generation does not need any specific requirement, and it is readily applicable in many structures, opening a new path for MEMS-based chaos generators.

Introduction

For half-a-century, chaos has triggered a lot of research around the world, both for fundamental and applied research. Chaos is characterized mainly by a non-periodic regime whose evolution is extremely sensitive to initial conditions. However, it is a deterministic system: an absolute knowledge of these initial conditions would enable to fully describe the system evolution, without the need to introduce any form of randomness. In practice, the knowledge of the initial conditions is limited by the precision of the measurement, such that the long-term prediction of the evolution of a chaotic system diverges, giving the illusion of a random system. This property is at the core of many researches, such as fluid mixing [1] or noiseless sensing [2].
Since the discovery of chaos synchronization [3], a tremendous amount of work has been dedicated to physical chaos generation, using either electronic [4] or optical [5] approaches. However, in the mechanical domain, chaos generation has mainly been limited to theoretical studies [6]–[8] since its experimental implementation is usually too complex, especially in micromechanical systems. Indeed, a common way to generate chaos consists in the realization of a bistable system, which is usually obtained by buckling the structure. However, the force required to perform the necessary buckling is typically generated through an electrostatic coupling, with an applied voltage ranging from a few tens to hundreds of volts [9], [10]. In this paper, we present an original way of generating chaos with a nonlinear non-buckled MEMS structure, requiring low voltages.

Methodology and results

We performed our experiment using a thin disk of radius of 400 µm and thickness of 10 µm. Using a piezoelectric transduction, the device is driven by applying a voltage between the bottom and the outer top electrode, and the mechanical displacement is then measured through the inner top electrode (Fig. 1 a). For small displacements, the device being in a linear regime, the amplitude varies proportionally with the driving force. The structure has a resonant frequency of 71.5 kHz and a quality factor of 1100 at low pressure (≈ 1 mbar). By increasing the driving force beyond the regime of small displacements, the MEMS will gradually enter in a nonlinear regime, mainly due to a cubic nonlinearity known as the Duffing nonlinearity. In this regime, for a positive nonlinearity, the shape of the resonance line bends towards higher frequencies, creating a hysteresis. In this frame, the resonator experiences a dynamical bistable regime similar to the one induced by buckling (Fig. 1 b).

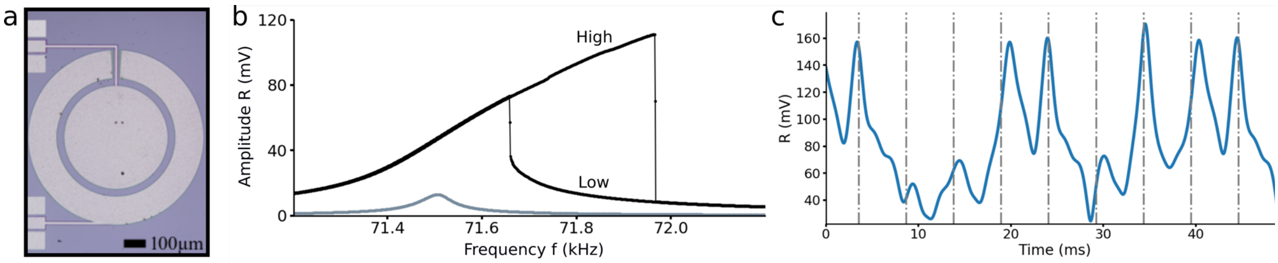

Figure 1: Dynamically bistable chaos. a) The microphotograph of the MEMS showing the outer and inner electrodes (the common bottom electrode covers the opposite side). **b)** At low amplitudes (grey), the MEMS response is linear, but as the drive amplitude increases, its resonance bends to form a hysteresis (black) with two available states (low and high amplitude) for a large frequency range. **c)** Applying a modulation on the driving signal, the system switches between the two states, and at a high enough modulation frequency the system's response displays a chaotic pattern. Grey dashed lines highlight the periodicity of the applied modulation, demonstrating no correlated periodicity in the output signal.

By driving the structure with an amplitude-modulated signal within its hysteresis, the system switches from one to the other state, namely high and low amplitude, and at a sufficiently high modulation frequency this switching becomes

erratic and the MEMS response exhibits a chaotic signal (Fig. 1 c). In our case, we used a modulation frequency of three times the bandwidth of the system, corresponding to 195 Hz.
Because of both non-periodic and non-reproducible features, specific tools are used to study the chaotic regime. In order to characterize the complexity of a chaotic regime, a common approach consists in a stroboscopic analysis of the generated signal, called the Poincaré section. This is performed by sampling a temporal signal at regular intervals defined by the modulation frequency (grey dashed lines in Fig. 1 c) and plotting the results in the phase space (Fig. 2 a). The generated Poincaré section presents the structure of the non-periodic chaos, extracting order from the apparent noise of the signal. Although a chaotic signal is unique, its Poincaré section represents a reproducible signature testifying the complexity of the generated chaos.

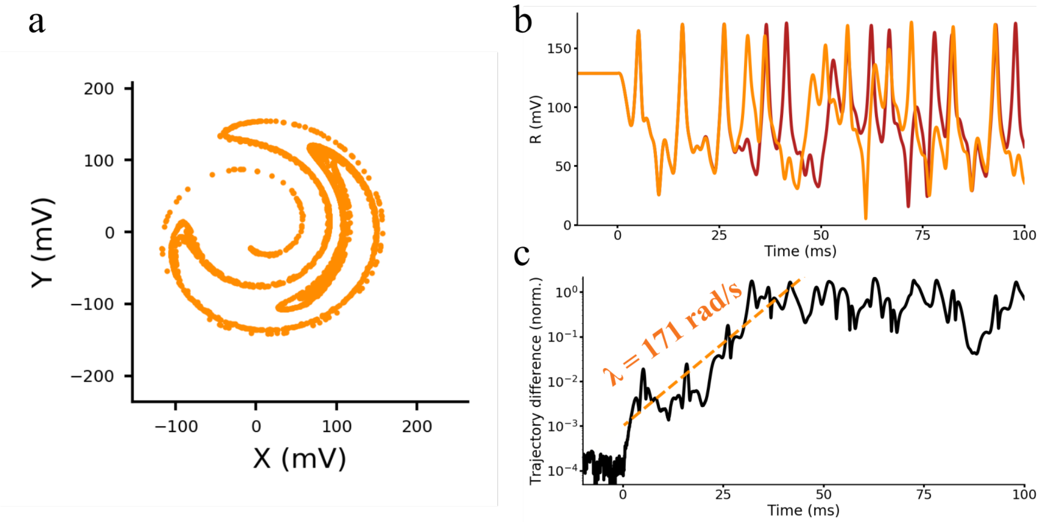

Figure 2: Poincaré section and Lyapunov exponent measurement. The chaotic signals are generated at a modulation frequency of 195 Hz with a driving voltage of 1 V. **a)** By sampling the chaotic signal every 1/195 second, a specific signature emerges from the chaos, forming a Poincaré section. **b)** In the chaotic regime, two measurements (orange and red) with extremely close initial conditions will tend to diverge in the chaotic regime (starting at the time $t = 0$). **c)** From the difference between the two trajectories, the Lyapunov exponent is extracted using an exponential fit.

Another interesting property of chaos lies in how sensitive to the initial conditions the system is. This property is characterized by the Lyapunov exponent, which describes how two initially close trajectories of the same system converge or diverge after some time. For a positive Lyapunov exponent, the trajectories diverge, which is the main property of chaotic systems, and its absolute value characterizes how fast the divergence is: it describes the memory of the system. The precision with which the Lyapunov exponent can be experimentally measured directly depends on how precisely the system can be set at similar initial conditions, ultimately limited by the noise of the system. In our case we were able to get initial conditions as close as 100 ppm, enabling to fit correctly the Lyapunov exponent, which we find to be 171 rad/s for our system (Fig. 2 b, c).

Conclusion

We demonstrated a new technique for MEMS-based chaos generation, using the dynamical bistability of the nonlinear system combined with an amplitude modulated driving force. We characterized the generated chaos with Poincaré sections and Lyapunov exponent measurements, giving information about the complexity and the memory of the system, essential to understand the behavior of the chaotic system.

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