Intense modal energy exchanges in a cantilever beam with a local geometrically nonlinear boundary condition: Simulation and experiment

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Summary. The aim of this research is to investigate and determine how a geometrically nonlinear boundary condition, i.e., a strong local nonlinearity, “redistributes” an input energy in the form of an impulsive load, among the modes of vibration of a cantilever beam, thereby increasing its efficacy of energy dissipation. The nonlinear boundary condition is created by grounding the free end of the cantilever beam through a linear spring-damper element at an angle relative to the neutral axis of the beam while at rest. By tracking the time-averaged energy and the effective instantaneous damping ratio of each mode of the beam, we numerically and experimentally show that there are sustained energy exchanges among the modes of vibration of the cantilever beam, the intensity of which depend on the degree of nonlinearity of the boundary condition.

Experimental setup and reduced-order finite-element modeling via system identification

Figures 1a and 1b depict the fully instrumented experimental apparatus consisting of the linear cantilever steel beam with Young’s modulus of 192 GPa, density of 7784 kg/m^3, cross-sectional area of 8×44.6 mm^2, length of 1.76 m, and the geometrically nonlinear boundary condition and its corresponding reduced-order model (ROM), respectively. Moreover, figure 1c shows a closeup of the geometrically nonlinear boundary condition, studied in [1], achieved by grounding the free end of the cantilever through a ¼ inch diameter steel rod whose bending stiffness and inherent damping provide the compliance and dissipation of the boundary condition.

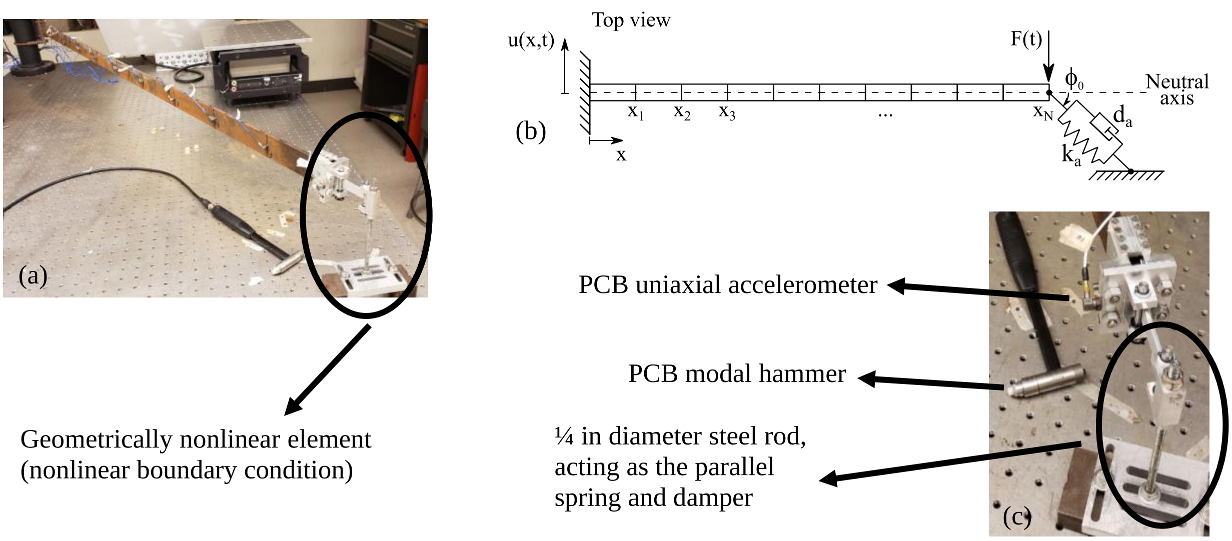

Figure1. Fully instrumented cantilever beam, grounded at its free end through a geometrically nonlinear boundary condition (a), the corresponding reduced-order model (ROM) (b), and zoomed-in view of the geometrically nonlinear boundary condition (c).

In order to create an accurate ROM of the apparatus, we implemented a two-step system identification approach: 1) By applying the Multi-input Multi-output Frequency Domain Identification (MFDID) [2] technique to the response of the cantilever beam, excited by an impulse as shown in figure 1b, we identified the modal parameters, i.e., natural frequencies and modal damping ratios, of the cantilever beam, uncoupled from the nonlinear attachment. 2) Next, we attached the nonlinear element to the fully identified cantilever beam and configured it such that $\phi_0 = 90^\circ$. Then, through time series optimization, we identified the unknown stiffness and damping parameters associated with the nonlinear attachment, k_a and d_a, after which we were able to accurately reproduce the experimentally measured response of the beam by the ROM, the governing equation of which can be expressed as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{f}_{\mathrm{nl}} = \mathbf{F}(\mathbf{t}), \qquad \mathbf{u}(0) = \mathbf{0}, \qquad \dot{\mathbf{u}}(0) = \mathbf{0}, \tag{1}$$

where $\mathbf{u} = [u_1, u_2, \dots, u_{\mathrm{N}}]^{\mathrm{T}}$ is the displacement vector corresponding to each node, x_i, $i = 1,2,\dots,\mathrm{N}$ (cf. figure 1b); $\mathbf{M}$, $\mathbf{K}$ are mass and stiffness matrices of the finite element model of the Euler-Bernoulli beam, and $\mathbf{C}$ is the proportional damping matrix of the beam, all obtained from the first step of the identification process. Moreover, $\mathbf{f}_{\mathrm{nl}}$, denotes the nonlinear force vector whose elements, except the N-th, are uniformly zero in time. The N-th element of $\mathbf{f}_{\mathrm{nl}}$, f_b, is expressed as

$$f_b = d_a\left[\frac{(x_{\mathrm{N}} + l_0 \sin\phi_0)^2}{(l_0\cos\phi_0)^2 + (x_{\mathrm{N}} + l_0\sin\phi_0)^2}\right]\dot{x}_{\mathrm{N}} + k_a(x_{\mathrm{N}} + l_0\sin\phi_0)\left[1 - \frac{l_0}{\sqrt{(l_0\cos\phi_0)^2 + (x_{\mathrm{N}} + l_0\sin\phi_0)^2}}\right], \tag{2}$$

where l_0 is the natural length of the nonlinear attachment.
It can be shown that the nonlinear force, (2), in the limit of $x_\mathrm{N} \ll l_0$ (low energy levels) can be linearized as

$$f_b|_{x_\mathrm{N} \ll l_0} = (d_a \sin^2 \phi_0)\dot{x}_\mathrm{N} + (k_a \sin^2 \phi_0)x_\mathrm{N}. \quad (3)$$

Combining (1)-(3) and assuming low energy, i.e., $x_\mathrm{N} \ll l_0$, we can construct the eigenvalue problem

$$\left[-\omega^2 \mathbf{M} + \left(\mathbf{K} + \mathbf{I}.\,\mathbf{f}_\mathrm{nl}|_{x_\mathrm{N} \ll l_0}\right)\right]\mathbf{u} = \mathbf{0}, \quad (4)$$

where $\mathbf{I}$ is the identity matrix, and consequently, find the so-called "linearized" modal basis of the system.

Modal energy scattering due to nonlinear boundary conditions

In order to compute the modal response of the nonlinear beam (both the experimental and computational models), we projected each measured response along the beam onto the linearized modal basis of the system. Following this we computed the time-averaged modal energies as described in [3]. By doing so, we can observe not only how much of the total energy of the beam is allocated to each mode, but also follow their variations with time and measure the maximum energy exchange among them for different angles of attachment, ϕ_0.
Finally, we defined a measure of energy exchange among modes by computing and comparing the maximum fluctuation in the percentage of each of the instantaneous modal energies, which corresponds to the maximum percentage of energy being exchanged among the modes. Figure 2 shows the energy exchange measure for the first two modes, for both the experimental system and the ROM.

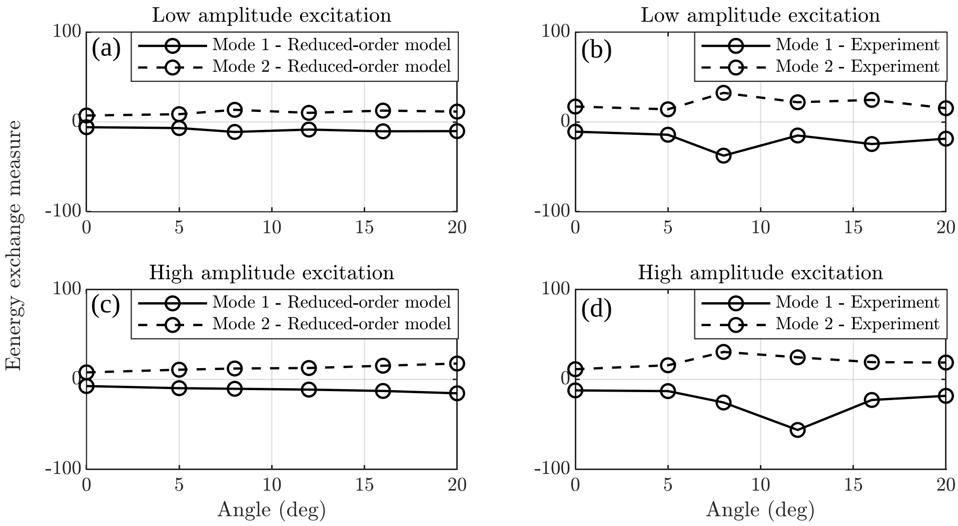

Figure 2. Energy exchange measures of the first two modes as a function of ϕ_0, computed from the ROM (a) and the experiment (b), for a low intensity impulse; (c) and (d) show similar graphs for a high intensity impulse.

Figure 2, especially the energy exchange measures computed from the experimental results, shows that, for a certain range of angle of inclination, $8^\circ < \phi_0 < 12^\circ$, the percentage of energy exchanged between modes 1 and 2, is maximized. This effect is achieved not only due to energy tunability of the nonlinearity but also due mainly to the existence of the angle of inclination, ϕ_0, which can be tuned to directly affect the degree of nonlinearity of the boundary condition.

Conclusions

In this research we investigated the effects of a strong local geometric nonlinearity on the modal interactions that occurred in a cantilever beam whose free end was grounded through a geometrically nonlinear element, which consisted of a linear spring-damper element at an angle, ϕ_0, relative to the neutral axis of the beam while at rest. After building an experimental apparatus, we created a reduced-order finite element model for the experiment and updated its parameters by implementing frequency domain and time domain system identification techniques. Next, we projected the response of the nonlinear beam onto the modal space obtained from the updated computational model, and calculated the amount of energy in each of the projected modes. We recorded the maximum energy exchange among the projected modes for different values of ϕ_0 and showed that by tuning this parameter properly it is possible to achieve the maximum amount of energy exchange from the low-frequency modes to high-frequency modes, thereby increasing the dissipation capabilities of the system.

References

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