Stability of Stationary Solution of Time Periodic Nonlinear Single DoF Time Delayed System Based on Impulse Response Function

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Summary. An extension of the semidiscretization method to impulse dynamic subspace (IDS) is summarized. This domain is the eigenspace of a measured impulse response function (IRF), which is commonly used in the industry. By considering the special properties of the measurement the stability of a single degree of freedom (DoF) milling model is presented as a representative example. Convergences are shown for Hopf and Period Doubling (PD) induced instabilities.

Introduction

In the verge of the new technological revolution more and more automatized solutions will appear in the daily life and very much in the manufacturing sector. One of the most difficult processes to be automatized are the ones that rely on human pattern recognition like understanding traffic situations. Dynamic characterization of machine tools are one of these problems, due to the special parameter identification techniques and selection methodologies used nowadays in the industry. In order to avoid that the impulse dynamic subspace (IDS, [1]) is used for carrying dynamic information.
In this work, a modeling technique is presented where the determination of model parameters is avoided and the process is directly described in the IDS. The main aim is to show that it is possible to derive theoretically the stability properties of a time-periodic milling process by only using the measurable IRF. The time-periodic nature of the milling process induces a stationary solution, which is always apparent and it directly determines the surface quality. However, this stationary solution can loose its stability by setting 'wrong' parameters, and can lead to a high amplitude, limiting oscillation. This limiting oscillation is mathematically stable, although the engineering jargon calls this as chatter instability [2], which refers mathematically to the unstable nature of the stationary time-periodic solution.
The main reason of this oscillation is the regenerative effect, when the consecutive tooth of the milling cutter cuts the surface left by the previous teeth. Since then many methods have been developed in time- and frequency- domain for determining stability of the corresponding stationary solution. Frequency domain solutions, like zeroth order approximation (ZOA, [2]) and multi-frequency (MF, [2]) solution, based on D-subdivision, and Hill's infinite determinant method, can include the measured frequency response functions (FRFs). However, this advantage comes with a huge disadvantage, namely, these methods only provide the critical (non-hyperbolic) limits and not actually the stability boundaries. Also to define the 'measure of stability' (distance from the border) is not straightforward in this case. One needs extremely specialized theorems to perform optimization. Time-domain methods like semidiscretization, time-finite element, collocation methods and spectral element methods, provide the Floquet-multipliers [3], whose magnitude are excellent to 'measure' stability for optimization purposes. However, all these methods rely on time-consuming modal parameter fitting, which computation time adds to the already slow extensive scanning of the parameter space constructing a given stability chart. In order to help on this disadvantage a method is proposed here by performing process modeling based on the IDS [4], which essentially a good candidate for avoiding manual fitting. Moreover by using time-domain based methods the 'measure' of stability is also granted by the magnitude of the multipliers.

Stationary Solution

There are plenty of papers dealing with modeling of regenerative milling processes [2, 5, 1]. In general, the milling process is not only time-periodic, but also nonlinear due to the degressive characteristic of the specific cutting force $f_{tra}(h)$ (N/m) [6] given in (tra) (figure $1a$) coordinate system. In milling, each ith ($i = 1, ..., Z$) tooth cuts different thickness of the workpiece h_i material during the rotation of the tool with angular velocity Ω. On the other hand, the chip thickness is also state dependent due to the regeneration [2], that is, $h_i(t) := h_i(t, x_t(\xi))$ ($\xi \in [-\tau, 0]$, $x_t(\xi) = x(t + \xi)$). In general, the resultant cutting force is time-periodic in its coefficients ($F_x(t, \bullet) = F_x(t + T, \bullet)$) and has the form (more detail in [6])

$$F_x(t, x_t(\xi)) := F_x(t, f_{tra}(h_i(t, , x_t(\xi)))) = F_{x,0}(t) + \Delta F_x(t, x_t(\xi)) + g_x(t, x_t(\xi)), \tag{1}$$

where the stationary part of the force is $F_{x,0}(t) = F_x(t, \overline{x}_t(\xi))$, while the linear variational part and the higher order terms are $\Delta F_x(t, x_t(\xi))$ and $g_x(t, x_t(\xi))$, respectively. The structural behavior of the machine tool is supposed to be linear, thus, it can be represented with an IRF as $h(\theta) = (\mathcal{F}^{-1}\{H(\omega)\})(\theta)$ subjected to the causality $h(\theta \leq 0) = 0$. If that is true, the response behaviour for a zero initial value can be represented by the Duhamel's integral as $\overline{x}_t(\theta) := \int_0^\theta h(\theta - \vartheta) F_x(t + \vartheta)\, \mathrm{d}\vartheta$. Since the stationary solution is time-periodic $\overline{x}_t(\theta) = \overline{x}_{t+T}(\theta) = \overline{x}_t(\theta + T)$, Duhamel's representation actually works for the nonlinear state-dependent forcing case too, if the stationary solution is considered frozen for the time period T in the interval $\theta \in [0, T]$. The stationary solution is then shifted with a sufficient enough transient time T_t to ensure periodicity and the boundary problem is solvable in both time and frequency domain with

$$\overline{x}_0(\theta) = \int_{-\infty}^{\theta} h(\theta - \vartheta) F_x(\vartheta, \overline{x}_\vartheta(\xi))\, \mathrm{d}\vartheta = \int_0^{T_\mathrm{t}+T} h(\theta + T_\mathrm{t} - \vartheta) F_x(\vartheta, \overline{x}_0((\xi + \vartheta) \bmod T))\, \mathrm{d}\vartheta. \tag{2}$$

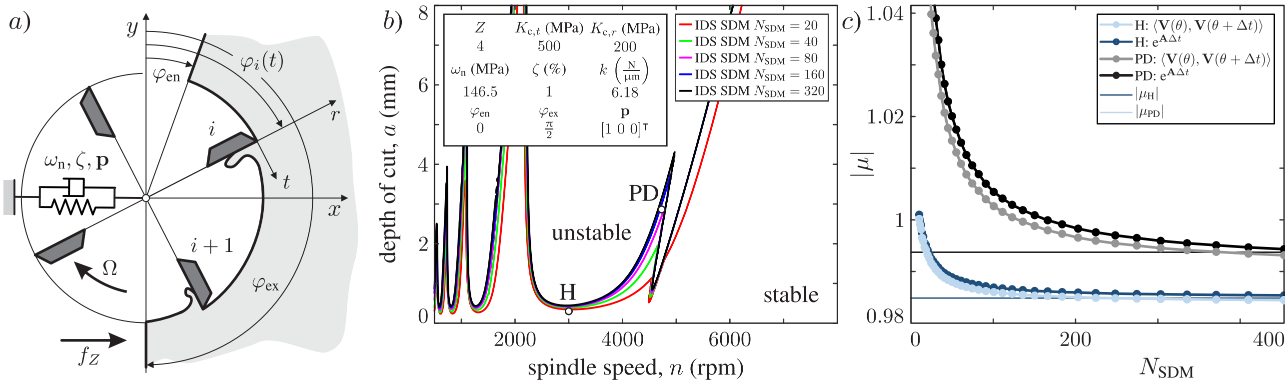

Figure 1: a) sketch of the milling model, b) stability charts with Hopf (H) and flip (PD) curves. Convergence for the solutions is checked in c) by varying the discretisation resolution N_{SDM} at H and PD points depicted in b).

Linear Stability of Stationary Solution

Perturbation is introduced around the time periodic stationary solution as $x = \overline{x} + u$. By neglecting the nonlinear terms g_x in (1), the linear variational system can be expressed in the following linear form

$$u_t(\theta) = \int_0^\infty G(\theta,\vartheta)\Delta F_u^-(t-\vartheta)\mathrm{d}\vartheta + \int_0^\theta h(\theta-\vartheta)\Delta F_x^+(t+\vartheta, u_{t+\vartheta}(\xi))\mathrm{d}\vartheta, \;\Rightarrow\; u_t(\theta) = u_{\mathrm{IF},t}(\theta) + u_{\mathrm{F0},t}(\theta, u_t(\xi)). \quad (3)$$

The first term u_{IF} of (3) represents the response for initial (variational) forcing ΔF_u^- (IF, alternative to initial condition), while the second term u_{F0} describes the solution for actual forcing $\Delta F_x(t, u_t(\xi)) = a\,K_{\mathrm{c}} A_x(t)(u(t) - u(t-\tau))$ (see (1)) combined with the transient solution for zero initial condition (F0). In the first term the so-called Green function can be replaced with the IRF as $G(\theta,\vartheta) := h(\theta+\vartheta) = \mathbf{V}(\theta)\mathbf{\Sigma}\,\mathbf{W}^{\mathsf{H}}(\vartheta)$, whose two left-singular-IRF for a single DoF system $\mathbf{V}(\theta) = [V_1(\theta)\ V_2(\theta)]$ can be used to introduce the new IDS as a result of SVD explained in [4]

$$u(t+\theta) = \mathbf{V}(\theta)\mathbf{q}(t), \quad \Rightarrow \quad \dot{\mathbf{q}}(t) = \mathbf{A}\,\mathbf{q}. \quad (4)$$

By defining the product $\langle \mathbf{a}(\xi), \mathbf{b}(\xi)\rangle := \int_0^\infty \mathbf{a}^{\mathsf{H}}(\xi)\mathbf{b}(\xi)\mathrm{d}\xi$ the system matrix can be derived as $\mathbf{A} = \langle \mathbf{V}(\theta), \mathbf{V}'(\theta)\rangle$ (using $\mathbf{V}'(\theta) = \langle G'(\theta,\vartheta), \mathbf{W}(\vartheta)\rangle\,\mathbf{\Sigma}^{-1}$, $G'(\theta,\vartheta) := h'(\theta+\vartheta)$, $h'(\theta) := (\mathcal{F}^{-1}\{\mathrm{i}\omega H(\omega)\})(\theta)$, [4]). Describing the behavior in the IDS ($\mathbf{q}_i = \mathbf{q}(t_i)$, $t_i = i\,\Delta t$, $\Delta\theta = \Delta t$) and considering that only physical displacement space $u_i = u(t_i)$ is needed for the calculation of the process force, the size of the problem can be significantly decreased. Thus

$$\mathbf{q}_{i+1} = \mathrm{e}^{\mathbf{A}\,\Delta t}\mathbf{q}_i + \langle \mathbf{V}(\theta), u_{\mathrm{F0},i+1}(\theta, u_i, \sigma_t(u_{i-l}))\rangle\,, \quad l = 0,1,2,...,r, \quad r = \left\lceil \tfrac{\tau}{\Delta\theta} - \tfrac{1}{2}\right\rceil + \left\lfloor \tfrac{p}{2}\right\rfloor, \quad (5)$$

where $u_{\mathrm{IF},i+1}(\theta) := u_{\mathrm{IF},t_i+\Delta t}(\theta) = \mathbf{V}(\theta+\Delta t)\mathbf{q}_i$, $u_{\mathrm{F0},i+1}(\theta, u_t(0), u_t(-\tau)) := u_{\mathrm{F0},t_i+\Delta t}(\theta, u_t(\xi)) = aK_{\mathrm{c}}\int_0^{\Delta t} h(\theta-\vartheta)A_x(t+\vartheta)(u_{t+\vartheta}(0) - u_{t+\vartheta}(-\tau))\mathrm{d}\vartheta$ and $u_t(0) := u_i$, $u_t(-\tau) \approx \sigma_t(u_{i-l}) = \sum_{k=0}^{p} P_k(t)\,u_{i-r+k}$. Using the homogeneous solution operator (exponential term in (5)) as $\mathrm{e}^{\mathbf{A}\,\Delta t} = \langle \mathbf{V}(\theta), \mathbf{V}(\theta+\Delta t)\rangle$ [4] the following semidiscretization map can be derived

$$\left.\begin{aligned} \mathbf{q}_{i+1} = \langle \mathbf{V}(\theta), \mathbf{V}(\theta+\Delta t)\rangle\,\mathbf{q}_i + \mathbf{D}_i\,u_i + ... + \mathbf{D}_{i-r+1}\,u_{i-r+1} + \mathbf{D}_{i-r}\,u_{i-r}, \\ u_{i+1} = \mathbf{V}(\Delta t)\mathbf{q}_i, \end{aligned}\right\}$$
$$\mathbf{D}_{i-l} = -aK_{\mathrm{c}}\left\langle \mathbf{V}(\theta), \int_0^{\Delta t} h(\theta-\vartheta)A_x(t_i+\vartheta)P_{r-l}(t_i+\vartheta)\mathrm{d}\vartheta\right\rangle, \quad P_r(t) = -1. \quad (6)$$

Conclusion

The map presented in (6) can be used to approximate the monodromy operator of the time-periodic milling system in order to calculate stability properties (figure 1c) of the corresponding stationary solution $\overline{x}$. This converging solution (figure 1c) uses the IDS which actually originated from measured IRF by using a well posed SVD on the homogeneous core of the dynamics ($G(\theta,\vartheta)$ at (3)). This theoretical framework can be extended for the entire period by facilitating larger portion of the corresponding IRF function by using nested convolutions.

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